## Lesson Objectives

### 9-1 Factors and Greatest Common Factors (pp. 474–479)
- Find prime factorizations of integers and monomials.
- Find the greatest common factors of integers and monomials.

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### 9-2 Factoring Using the Distributive Property (pp. 480–486)
*Preview:* Use algebra tiles and a product mat to factor binomials.
- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

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<td>9-2 Factoring Using the Distributive Property (with 9-2 Preview)</td>
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### 9-3 Factoring Trinomials: $x^2 + bx + c$ (pp. 487–494)
*Preview:* Use algebra tiles to factor trinomials.
- Factor trinomials of the form $x^2 + bx + c$.
- Solve equations of the form $x^2 + bx + c = 0$.

<table>
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### 9-4 Factoring Trinomials: $ax^2 + bx + c$ (pp. 495–500)
- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

<table>
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### 9-5 Factoring Differences of Squares (pp. 501–506)
- Factor binomials that are the differences of squares.
- Solve equations involving the differences of squares.

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<td>9-5 Factoring Differences of Squares</td>
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### 9-6 Perfect Squares and Factoring (pp. 508–514)
- Factor perfect square trinomials.
- Solve equations involving perfect squares.

<table>
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<tbody>
<tr>
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<td>9-6 Perfect Squares and Factoring</td>
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### Study Guide and Practice Test (pp. 515–519)
- 1 day for Basic/ Average
- 1 day for Advanced

### Standardized Test Practice (pp. 520–521)
- 1 day for Basic/ Average
- 0.5 day for Advanced

### Chapter Assessment
- 1 day for Basic/ Average
- 1 day for Advanced
- 0.5 day for Basic/ Average
- 0.5 day for Advanced

### TOTAL
- 14 days for Regular
- 12 days for Basic/ Average
- 8 days for Advanced
- 6 days for Basic/ Average

*Pacing suggestions for the entire year can be found on pages T20–T21.*
# Chapter Resource Manager

## Chapter 9 Resource Masters

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<td>(Preview: algebra tiles, product mat)</td>
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</table>

*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

ELL Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Factors and Greatest Common Factors

The factors of a given number are all the numbers that divide the number evenly. This includes the number itself and 1. The factors of a number can be found by determining all the pairs of numbers whose product is that number. Natural numbers greater than 1 are classified as either prime or composite. Prime numbers have exactly two factors while composite numbers have more than two factors. The number 1 is neither prime nor composite. A prime factorization is the expression of a number as the product of factors that are all prime numbers. The prime factorization of a negative integer is expressed as the product of prime numbers. Monomials can be written in factored form. A monomial in factored form is the product of prime numbers and variables. Variables, however, cannot have an exponent greater than 1. So \( x^3 \) must be written as \( x \cdot x \cdot x \) in factored form.

Prime factorizations are used to determine the greatest common factor (GCF) of two or more integers or monomials. The GCF is the product of all the common prime factors of the two integers or monomials. If 1 is the only common factor, then they are relatively prime.

Factoring Using the Distributive Property

Many polynomials also have factors. Some polynomials are the product of a polynomial and a monomial. Reverse the process of multiplying a polynomial by a monomial to factor using the Distributive Property. First find the greatest common factor of the terms of the polynomial. If the GCF is not 1, then rewrite each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

If a polynomial has four or more terms you can factor by grouping. Group terms in pairs that have common factors. Use the Distributive Property to factor the GCF from each pair of terms. The binomials in each pair of factored terms should be identical. Use the Distributive Property to factor out the common binomial factor. The remaining factors are grouped to form a second binomial. It may appear that the factored pairs do not have identical binomials, but one may be the additive inverse of the other. Write one as the product of \(-1\) and its additive inverse. Then multiply the GCF of that pair by \(-1\).

If the product of two factors is 0, then at least one of them is 0 according to the Zero Product Property. If an equation has the form \( ab = 0 \) or can be written in this form by factoring, then the Zero Product Property can be applied to solve the equation. Set each factor equal to 0 and solve each resulting equation.

Prior Knowledge
Students studied prime numbers and greatest common factors in previous courses. They also found the prime factorization of numbers. In Chapter 8, students learned the rules for dividing monomials.

This Chapter
This chapter covers the factoring of integers, monomials and polynomials. Students learn to find the greatest common factor of the monomials in a polynomial. They use the GCF and the Distributive Property to factor polynomials. Students also apply the Zero Product Property to solve quadratic equations.

Future Connections
Factoring polynomials is used to solve many real-world problems. It is basic to studying more about polynomial equations and functions.
9-3 **Factoring Trinomials:** 
\[ x^2 + bx + c \]

The FOIL method was used to multiply two binomials. Reverse the FOIL method to factor a quadratic polynomial of the form \( x^2 + bx + c \) into two binomials. Find two numbers \( m \) and \( n \) whose product is \( c \) and whose sum is \( b \). The two numbers are the last terms of the two binomials \( (x + m) \) and \( (x + n) \).

If \( b \) is negative and \( c \) is positive then \( m \) and \( n \) must both be negative. If \( c \) is negative, then \( m \) and \( n \) must have different signs. This is because the product of two numbers with different signs is negative.

The Zero Product Property can be used to solve some quadratic equations written in the form \( x^2 + bx + c = 0 \). Factor the trinomial, and then set each factor equal to 0. Solve each equation to find the solution of the quadratic equation. Be sure to check the solutions in the original equation.

Other aspects of factoring to watch for are factoring out a common factor, applying a technique more than once, and applying several techniques. Use any of the appropriate techniques and the Zero Product Property to solve many polynomial equations.

9-6 **Perfect Squares and Factoring**

Some trinomials have patterns that make their factoring easier. In Lesson 8-8, students learned about patterns for the square of a sum, \( (a + b)^2 = a^2 + 2ab + b^2 \), and the square of a difference, \( (a - b)^2 = a^2 - 2ab + b^2 \). These products, \( a^2 + 2ab + b^2 \) and \( a^2 - 2ab + b^2 \), are called perfect square trinomials, because they are the result of squaring a binomial. To recognize a perfect square trinomial, first determine if the first and last terms are perfect squares. Then find the square roots of the first and last terms, checking to see if twice the product of these square roots is equal to the middle term of the trinomial. If the trinomial is a perfect square, and the middle term is positive use the pattern \( a^2 + 2ab + b^2 = (a + b)^2 \) to factor it. If the middle term is negative, use the pattern \( a^2 - 2ab + b^2 = (a - b)^2 \). It is important to note that the last term of a perfect square trinomial cannot be negative.

If one side of an equation is a perfect square or can be written as a perfect square, then the Square Root Property can be applied to solve the equation. The Square Root Property allows you to take the square root of each side of an equation, so long as both the positive and negative square roots of a number are taken into account. So, for any number \( n \) that is greater than 0, if \( x^2 = n \), then \( x = \pm \sqrt{n} \). Two solutions result from such equations, one using the positive square root and one using the negative square root.

9-4 **Factoring Trinomials:**
\[ ax^2 + bx + c \]

To factor a trinomial in which the coefficient of \( x^2 \) is not 1, first check to see if the terms of the polynomial have a GCF. If so, factor it out. If the coefficient of \( x^2 \) is still not 1, or there is no GCF, factor \( ax^2 + bx + c \) by making an organized list of the factors of the product of \( a \) and \( c \). For example, to factor \( 8x^2 - 5x - 3 \), make an organized list of the factors of \( 8 \cdot (-3) \), or \(-24\). Look for a pair of factors, \( m \) and \( n \), whose sum is equal to \( b \) in the trinomial. Then rewrite the trinomial, replacing \( bx \) with \( mx + nx \). The new polynomial has four terms. Use the factoring by grouping technique shown in Lesson 9-2 to factor this polynomial into two binomial factors. A polynomial that cannot be factored is a prime polynomial. Use the above method and the Zero Product Property to solve equations in the form \( ax^2 + bx + c = 0 \).

9-5 **Factoring Differences of Squares**

Lesson 8-8 discussed the pattern for the product of a sum and a difference: \( (a + b)(a - b) = a^2 - b^2 \). The binomial \( a^2 - b^2 \) is called the difference of two squares and can be factored as the product of a sum and a difference: \( a^2 - b^2 = (a - b)(a + b) \). To do this, identify \( a \) and \( b \), the square roots of the first and last terms, respectively. Then apply the pattern.
### Key to Abbreviations

- **TWE** = Teacher Wraparound Edition
- **CRM** = Chapter Resource Masters

### Additional Intervention Resources

- **The Princeton Review’s Cracking the SAT & PSAT**
- **The Princeton Review’s Cracking the ACT**
- **ALEKS**

### TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

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### Ongoing

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<td>Practice Quiz 1, p. 486</td>
<td><em>Prerequisite Skills Workbook</em>, pp. 13–14</td>
<td><a href="http://www.algebra1.com/self_check_quiz">www.algebra1.com/self_check_quiz</a></td>
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### Mixed Review

| Type          | pp. 479, 486, 494, 500, 506, 514                                              | Cumulative Review, *CRM* p. 576                                                  |                                                          |

### Error Analysis

| Type          | Find the Error, pp. 492, 498, 504                                              | Find the Error, *TWE* pp. 492, 498, 504                                          |                                                          |
|               | Common Misconceptions, pp. 483, 502                                              | Unlocking Misconceptions, *TWE* p. 497                                            |                                                          |
|               |                                                                                | Tips for New Teachers, *TWE* pp. 490, 509                                         |                                                          |

### Standardized Test Practice

| Type          | pp. 479, 486, 494, 500, 503, 505, 514, 519, 520–521                             | *TWE* pp. 520–521                                                                 | Standardized Test Practice                                |
|               |                                                                                | Standardized Test Practice, *CRM* pp. 577–578                                     | [CD-ROM](http://www.algebra1.com/standardized_test)      |
|               |                                                                                |                                                                                 |                                                          |

### Open-Ended Assessment

| Type          | Writing in Math, pp. 479, 485, 494, 500, 506, 514                             | Modeling: *TWE* pp. 479, 506                                                      |                                                          |
|               | Open Ended, pp. 477, 484, 492, 498, 504, 512                                | Speaking: *TWE* pp. 486, 494                                                      |                                                          |
|               | Standardized Test, p. 521                                                     | Writing: *TWE* pp. 500, 514                                                       |                                                          |
|               |                                                                                | Open-Ended Assessment, *CRM* p. 571                                              |                                                          |

### Chapter Assessment

| Type          | Study Guide, pp. 515–518                                                       | Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 559–564                      | TestCheck and Worksheet Builder                           |
|               | Practice Test, p. 519                                                          | Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 565–570                        | (see below)                                               |

**TestCheck and Worksheet Builder**

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.
Intervention Technology

AlgePASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

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<td>26 Solving Quadratic Equations Using Factoring</td>
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<tr>
<td>9-6</td>
<td>27 Factoring Expressions I</td>
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ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student’s needs. Subscribe at www.k12aleks.com.

Intervention at Home

Parent and Student Study Guide
Parents and students may work together to reinforce the concepts and skills of this chapter. (Workbook, pp. 68–74 or log on to www.algebra1.com/parent_student)

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra1.com/extra_examples
  www.algebra1.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra1.com/vocabulary_review
  www.algebra1.com/chapter_test
  www.algebra1.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 1 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 473
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 477, 484, 492, 498, 504, 512)
- Reading Mathematics, p. 507
- Writing in Math questions in every lesson, pp. 479, 485, 494, 500, 506, 514
- Reading Study Tip, pp. 489, 511
- WebQuest, p. 479

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 473, 515
- Study Notebook suggestions, pp. 477, 480, 484, 488, 492, 498, 504, 507, 512
- Modeling activities, pp. 479, 506
- Speaking activities, pp. 486, 494
- Writing activities, pp. 500, 514
- Differentiated Instruction, (Verbal/Linguistic), p. 475
- ELL Resources, pp. 472, 475, 478, 485, 493, 499, 505, 507, 513, 515

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (Chapter 9 Resource Masters, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (Chapter 9 Resource Masters, pp. 527, 533, 539, 545, 551, 559)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- Teaching Mathematics with Foldables provides suggestions for promoting cognition and language.
- Reading and Writing in the Mathematics Classroom
- WebQuest and Project Resources
- Hot Words/Hot Topics Sections 1.2, 3.2, 6.2

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

The factoring of polynomials can be used to solve a variety of real-world problems and lays the foundation for the further study of polynomial equations. Factoring is used to solve problems involving vertical motion. For example, the height \( h \) in feet of a dolphin that jumps out of the water traveling at 20 feet per second is modeled by a polynomial equation. Factoring can be used to determine how long the dolphin is in the air. You will learn how to solve polynomial equations in Lesson 9-2.

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 9 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 9 test.

**Key Vocabulary**
- factored form (p. 475)
- factoring by grouping (p. 482)
- prime polynomial (p. 497)
- difference of squares (p. 501)
- perfect square trinomials (p. 508)
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lessons 9-2 through 9-6  Distributive Property
Rewrite each expression using the Distributive Property. Then simplify.
(For review, see Lesson 1-5.)
1. 3(4 − x) 2. a(a + 5) 3. −7(n² − 3n + 1) 4. 6y(−3y − 5y² + y³)
   12 − 3x  a² + 5a  −7n² + 21n − 7  −18y² − 30y³ + 6y⁴

For Lessons 9-3 and 9-4  Multiplying Binomials
Find each product.
(For review, see Lesson 8-7.)
5. (x + 4)(x + 7) 6. (3n − 4)(n + 5) 7. (6a − 2b)(9a + b) 8. (−x − 8y)(2x − 12y)
x² + 11x + 28 3n² + 11n − 20 54a² − 12ab − 2b²  −2x² − 4xy + 96y²

For Lessons 9-5 and 9-6  Special Products
Find each product.
(For review, see Lesson 8-8.)
9. (y + 9)² 10. (3a − 2)² 11. (n − 5)(n + 5) 12. (6p + 7q)(6p − 7q)
y² + 18y + 81 9a² − 12a + 4  n² − 25  36p² − 49q²

For Lesson 9-6  Square Roots
Find each square root.
(For review, see Lesson 2-7.)
13. √121  14. √0.0064  0.08  15. √25  5/6  16. √98  2/7

Prerequisite Skills  The following sections provide a review of the basic concepts needed before beginning Chapter 9. Page references are included for additional student help. Additional review is provided in the Prerequisite Skills Workbook, pp. 13–14.

For Lesson  | Prerequisite Skill
-----------|-----------------|
9-2    | Distributive Property, p. 479
9-3    | Multiplying Polynomials, p. 486
9-4    | Factoring by Grouping, p. 494
9-5    | Square Roots, p. 500
9-6    | Special Products, p. 506

Organization of Data and Questioning  Before beginning each lesson, ask students to think of one question that comes to mind as they skim through the lesson. Write the question on the front of the corresponding lesson tab. As students read and work through the lesson, ask them to record the answer to their question under the tab. Students can also use their Foldables to take notes, record concepts, define terms, and record other questions that arise about factoring.
**Focus**

**5-Minute Check Transparency 9-1** Use as a quiz or review of Chapter 8.

Mathematical Background notes are available for this lesson on p. 472C.

How are prime numbers related to the search for extraterrestrial life?

Ask students:

- What is a prime number?
  A prime number is any whole number, greater than one, whose only factors are one and itself.

- Why might a radio signal from space composed of prime numbers be significant?
  Sample answer: A signal composed of only prime numbers would seem to signify that it was sent by intelligent beings.

**Vocabulary**
- prime number
- composite number
- prime factorization
- factored form
- greatest common factor (GCF)

**What You’ll Learn**

- Find prime factorizations of integers and monomials.
- Find the greatest common factors of integers and monomials.

**How are prime numbers related to the search for extraterrestrial life?**

In the search for extraterrestrial life, scientists listen to radio signals coming from faraway galaxies. How can they be sure that a particular radio signal was deliberately sent by intelligent beings instead of coming from some natural phenomenon?

What if that signal began with a series of beeps in a pattern comprised of the first 30 prime numbers (“beep-beep,” “beep-beep-beep,” and so on)?

**PRIME FACTORIZATION** Recall that when two or more numbers are multiplied, each number is a factor of the product. Some numbers, like 18, can be expressed as the product of different pairs of whole numbers. This can be shown geometrically. Consider all of the possible rectangles with whole number dimensions that have areas of 18 square units.

```
    1 x 18
    2 x 9
    3 x 6
```

The number 18 has 6 factors, 1, 2, 3, 6, 9, and 18. Whole numbers greater than 1 can be classified by their number of factors.

### Key Concept: Prime and Composite Numbers

<table>
<thead>
<tr>
<th>Words</th>
<th>Examples</th>
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<tbody>
<tr>
<td>A whole number, greater than 1, whose only factors are 1 and itself, is called a <strong>prime number</strong>.</td>
<td>2, 3, 5, 7, 11, 13, 17, 19</td>
</tr>
<tr>
<td>A whole number, greater than 1, that has more than two factors is called a <strong>composite number</strong>.</td>
<td>4, 6, 8, 9, 10, 12, 14, 15, 16, 18</td>
</tr>
</tbody>
</table>

0 and 1 are neither prime nor composite.

### Example 1 Classify Numbers as Prime or Composite

Factor each number. Then classify each number as **prime** or **composite**.

**a. 36**

To find the factors of 36, list all pairs of whole numbers whose product is 36.

```
1 x 36
2 x 18
3 x 12
4 x 9
6 x 6
```

Therefore, the factors of 36, in increasing order, are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Since 36 has more than two factors, it is a composite number.

### Study Tip

**Listing Factors**

Notice that in Example 1, 6 is listed as a factor of 36 only once.

474 Chapter 9 Factoring

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**Resource Manager**

**Workbook and Reproducible Masters**

*Chapter 9 Resource Masters*
- Study Guide and Intervention, pp. 523–524
- Skills Practice, p. 525
- Practice, p. 526
- Reading to Learn Mathematics, p. 527
- Enrichment, p. 528

*Parent and Student Study Guide Workbook*, p. 68

*Prerequisite Skills Workbook*, pp. 13–14

**Transparencies**

- 5-Minute Check Transparency 9-1
- Answer Key Transparencies

**Technology**

Interactive Chalkboard
b. 23

The only whole numbers that can be multiplied together to get 23 are 1 and 23. Therefore, the factors of 23 are 1 and 23. Since the only factors of 23 are 1 and itself, 23 is a prime number.

When a whole number is expressed as the product of factors that are all prime numbers, the expression is called the prime factorization of the number.

Example 2 Prime Factorization of a Positive Integer

Find the prime factorization of 90.

Method 1

90 = 2 \cdot 45

= 2 \cdot 3 \cdot 15

= 2 \cdot 3 \cdot 3 \cdot 5

The least prime factor of 90 is 2.

All of the factors in the last row are prime. Thus, the prime factorization of 90 is 2 \cdot 3 \cdot 3 \cdot 5.

Method 2

Use a factor tree.

90

\[ \begin{array}{c}
90 = 9 \cdot 10 \\
90 = 9 \cdot 3 \cdot 10 \\
90 = 9 \cdot 3 \cdot 2 \cdot 5 \\
90 = 3 \cdot 3 \cdot 2 \cdot 5
\end{array} \]

All of the factors in the last branch of the factor tree are prime. Thus, the prime factorization of 90 is 2 \cdot 3 \cdot 3 \cdot 5 or 2 \cdot 3^2 \cdot 5.

A negative integer is factored completely when it is expressed as the product of -1 and prime numbers.

Example 3 Prime Factorization of a Negative Integer

Find the prime factorization of -140.

\[ \begin{array}{c}
-140 = -1 \cdot 140 \\
= -1 \cdot 2 \cdot 70 \\
= -1 \cdot 2 \cdot 2 \cdot 35 \\
= -1 \cdot 2^2 \cdot 5 \cdot 7
\end{array} \]

Thus, the prime factorization of -140 is -1 \cdot 2^2 \cdot 5 \cdot 7 or -1 \cdot 2^2 \cdot 5 \cdot 7.

A monomial is in factored form when it is expressed as the product of prime numbers and variables and no variable has an exponent greater than 1.
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In-Class Example

Factor each monomial completely.

4. \(18x^3y^2\)

5. \(-26rst^2 - 1 \cdot 2 \cdot 13 \cdot r \cdot s \cdot t \cdot t\)

Teaching Tip: Remind students that prime factorization of a constant can have exponents greater than 1, but prime factorization of variable values cannot.

GREATEST COMMON FACTOR

5. Find the GCF of each set of monomials.

a. 12 and 18  The GCF is 6

b. 27a^2b and 15ab^2c  The GCF is 3ab

6. CRAFTS: Rene has crocheted 32 squares for an afghan. Each square is 1 foot square. She is not sure how she will arrange the squares but does know it will be rectangular and have a ribbon trim. What is the maximum amount of ribbon she might need to finish the afghan? 66 ft

Example 4  Prime Factorization of a Monomial

Factor each monomial completely.

a. \(12a^2b^3\)

\[12a^2b^3 = 2 \cdot 6 \cdot a \cdot a \cdot b \cdot b \cdot b\]

Thus, \(12a^2b^3\) in factored form is \(2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b\).

b. \(-66pq^2\)

\[-66pq^2 = -1 \cdot 66 \cdot p \cdot q \cdot q\]

Express -66 as -1 times 66.

\[= -1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q\]

\[= -1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q\]

Thus, \(-66pq^2\) in factored form is \(-1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q\).

GREATEST COMMON FACTOR  Two or more numbers may have some common prime factors. Consider the prime factorization of 48 and 60.

\[48 = 2 \cdot 2 \cdot 2 \cdot 3\]  Factor each number.

\[60 = 2 \cdot 2 \cdot 3 \cdot 5\]  Circle the common prime factors.

The integers 48 and 60 have two 2s and one 3 as common prime factors. The product of these common prime factors, 2 \cdot 2 \cdot 3 or 12, is called the greatest common factor (GCF) of 48 and 60. The GCF is the greatest number that is a factor of both original numbers.

Key Concept  Greatest Common Factor (GCF)

- The GCF of two or more integers is the product of the prime factors common to the integers.
- The GCF of two or more monomials is the product of their common factors when each monomial is in factored form.
- If two or more integers or monomials have a GCF of 1, then the integers or monomials are said to be relatively prime.

Example 5  GCF of a Set of Monomials

Find the GCF of each set of monomials.

a. 15 and 16

\[15 = 3 \cdot 5\]  Factor each number.

\[16 = 2 \cdot 2 \cdot 2 \cdot 2\]  Circle the common prime factors, if any.

There are no common prime factors, so the GCF of 15 and 16 is 1. This means that 15 and 16 are relatively prime.

b. 36x^2y and 54xy^2z

\[36x^2y = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot y\]  Factor each number.

\[54xy^2z = 2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot y \cdot z\]  Circle the common prime factors.

The GCF of 36x^2y and 54xy^2z is 2 \cdot 3 \cdot 3 \cdot x \cdot y or 18xy.

Interactive Chalkboard

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

• Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
• Additional, Your Turn exercises for each example
• The 5-Minute Check Transparencies
• Hot links to Glencoe Online Study Tools

Teacher to Teacher

Lisa Cook  Kaysville Jr. H.S., Kaysville, UT

“To help in identifying prime factors, I like to have my students explore the Sieve of Eratosthenes. Use a 10-by-10 grid with the numbers 1-100 on it. Cross out 1 since prime numbers are greater than 1. Circle 2 and cross out all multiples of 2. Circle 3 and cross out multiples of 3. Continue with the next odd number until all multiples have been eliminated. The circled numbers are the prime numbers less than 100.”
Example 6 Use Factors

The area of a rectangle is 28 square inches. If the length and width are both whole numbers, what is the maximum perimeter of the rectangle?

Find the factors of 28, and draw rectangles with each length and width. Then find each perimeter.

The factors of 28 are 1, 2, 4, 7, 14, and 28.

The greatest perimeter is 58 inches. The rectangle with this perimeter has a length of 28 inches and a width of 1 inch.

Check for Understanding

Concept Check

1. Determine whether the following statement is true or false. If false, provide a counterexample. All prime numbers are odd. false; 2
2. Explain what it means for two numbers to be relatively prime. Their GCF is 1.
3. OPEN ENDED Name two monomials whose GCF is 5x^2.

Guided Practice

Find the factors of each number. Then classify each number as prime or composite.

4. 8; composite 5. 17; prime 6. 112; composite

Find the prime factorization of each integer.

7. 45; 3^2 · 5 8. −32; −1 · 2^5 9. −150; −1 · 2 · 3 · 5^2

Factor each monomial completely.

10. 4p^2 · 2 · p · p 11. 36b^3 · 2^3 · 3 · 13 · b · b · c · c 12. −100x^2yz^2

Find the GCF of each set of monomials.

13. 10, 15 14. 18xy, 36y^2 18y 15. 54, 63, 180

16. 25n, 21m 17. 12x^2b, 90a^2b^2c 6a^2b 18. 15p^2, 35s^2, 70rs

Application

19. GARDENING Ashley is planting 120 tomato plants in her garden. In what ways can she arrange them so that she has the same number of plants in each row, at least 5 rows of plants, and at least 5 plants in each row? 5 rows of 24 plants, 6 rows of 20 plants, 8 rows of 15 plants, or 10 rows of 12 plants

Practice and Apply

Find the factors of each number. Then classify each number as prime or composite.


www.algebra1.com/self_check_quiz

Answers

20. 1, 19; prime 21. 1, 5, 25; composite 22. 1, 2, 4, 5, 8, 10, 16, 20, 40, 80; composite 23. 1, 61; prime 24. 1, 7, 13, 91; composite 25. 1, 7, 17, 119; composite 26. 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126; composite 27. 1, 2, 4, 8, 16, 19, 38, 76, 152, 304; composite

Lesson 9-1 Factors and Greatest Common Factors 477
## Factoring

### Problem Solving Strategy

1. **What is the greatest known prime number?**
   - **Solution:** The greatest known prime number as of my knowledge is 28,982,368,271, but this number may change as new prime numbers are discovered.

2. **What is the maximum perimeter of the rectangle?**
   - **Solution:** The maximum perimeter of a rectangle is achieved when it is a square. If the side length is $s$, the perimeter is $P = 4s$. Therefore, the maximum perimeter is $4 	imes 96 = 384$ mm.

3. **What is the maximum size of square tile Ms. Baxter can use and not have to cut any of the sides?**
   - **Solution:** The maximum size of a square tile that Ms. Baxter can use is determined by the greatest common divisor (GCD) of the dimensions of the floor. If the dimensions of the floor are $30$ mm and $12$ mm, the maximum size of a square tile is $2$ mm.

4. **What is the prime factorization of each integer?**
   - **Examples:**
     - $120 = 2^3 	imes 3 	imes 5$  
     - $180 = 2^2 	imes 3^2 	imes 5$

5. **Find the prime factorization of each integer.**
   - **Examples:**
     - $440 = 2^3 	imes 5 	imes 11$  
     - $1260 = 2^2 	imes 3^2 	imes 5 	imes 7$

### More About...

#### Marching Bands

Drum Corps International (DCI) is a nonprofit youth organization serving junior drum and bugle corps around the world. Members of these marching bands range from 14 to 21 years of age. Source: www.dci.org

### Homework Help

**For Exercises:**
- **Examples:**
  - 20–27, 62
  - 32–39
  - 40–47
  - 48–61
  - 62–79

**Extra Practice:**
- See page 839.

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### GEOMETRY

For Exercises 28 and 29, consider a rectangle whose area is 96 square millimeters and whose length and width are both whole numbers.

28. **What is the minimum perimeter of the rectangle?**
   - **Solution:** The minimum perimeter of a rectangle is achieved when it is a square. If the side length is $96$ mm, the perimeter is $P = 4 	imes 96 = 384$ mm.

29. **What is the maximum perimeter of the rectangle?**
   - **Solution:** The maximum perimeter of a rectangle is achieved when it is a square. If the side length is $96$ mm, the perimeter is $P = 4 	imes 96 = 384$ mm.

### COOKIES

For Exercises 30 and 31, use the following information.

A bakery packages cookies in two sizes of boxes, one with 18 cookies and the other with 24 cookies. Both packages must be wrapped in cellophane before they are placed in a box. To save money, the bakery will use the same size cellophane packages for each box.

30. **How many cookies should the bakery place in each cellophane package to maximize the number of cookies in each package?**
   - **Solution:** The maximum number of cookies is obtained when the number of cookies per package is the greatest common divisor (GCD) of the two package sizes. The GCD of 18 and 24 is 6, so the bakery should package 6 cookies per cellophane.

31. **How many cellophane packages will go in each size box?**
   - **Solution:** The number of packages is determined by the number of cookies divided by the number of cookies per package. For 18 cookies, the number of packages is $18 / 6 = 3$, and for 24 cookies, the number of packages is $24 / 6 = 4$.

---

### Answers

28. The factors of 96 whose sum when doubled is the least are 12 and 18.

29. The factors of 96 whose sum when doubled is the greatest are 1 and 96.
67. **GEOMETRY** The area of a triangle is 20 square centimeters. What are possible whole-number dimensions for the base and height of the triangle? **See margin.**

68. **CRITICAL THINKING** Suppose 6 is a factor of ab, where a and b are natural numbers. Make a valid argument to explain why each assertion is true or provide a counterexample to show that an assertion is false.
   a. 6 must be a factor of a or of b. **True; see margin for explanation.**
   b. 3 must be a factor of a or of b. **True; see margin for explanation.**
   c. 3 must be a factor of a and of b. **False; counterexample: a = 3, b = 1082**

69. **Writing in Math** Answer the question that was posed at the beginning of the lesson. **See p. 521A.**

How are prime numbers related to the search for extraterrestrial life?

Include the following in your answer:
- a list of the first 30 prime numbers and an explanation of how you found them, and
- an explanation of why a signal of this kind might indicate that an extraterrestrial message is to follow.

70. Miko claims that there are at least four ways to design a 120-square-foot rectangular space that can be tiled with 1-foot by 1-foot tiles. Which statement best describes this claim? **D**
   - A. Her claim is false because 120 is a prime number.
   - B. Her claim is false because 120 is not a perfect square.
   - C. Her claim is true because 240 is a multiple of 120.
   - D. Her claim is true because 120 has at least eight factors.

71. Suppose \( \Psi_x \) is defined as the largest prime factor of x. For which of the following values of x would \( \Psi_x \) have the greatest value? **A**
   - A. 53
   - B. 74
   - C. 99
   - D. 117

72. **WebQuest** The area of the solar system. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

**Answers**

40. 2 \cdot 3 \cdot 11 \cdot d \cdot d \cdot d
41. 5 \cdot 17 \cdot x \cdot y \cdot y
42. 7 \cdot 7 \cdot a \cdot a \cdot b \cdot b
43. 2 \cdot 5 \cdot 5 \cdot g \cdot h
44. 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot p \cdot q \cdot q
45. 3 \cdot 3 \cdot 3 \cdot 3 \cdot n \cdot n \cdot m
46. \left(1, 1 \cdot 3 \cdot 61 \cdot x \cdot y \cdot z \cdot z \cdot zight)
47. \left(1, 1 \cdot 13 \cdot 13 \cdot a \cdot b \cdot c \cdot cight)
48. base 1 cm, height 40 cm; base 2 cm, height 20 cm; base 4 cm, height 10 cm; base 5 cm, height 8 cm; base 8 cm, height 5 cm; base 10 cm, height 4 cm; base 20 cm, height 2 cm; base 40 cm, height 1 cm
68b. If 6 is a factor of ab, then the prime factorization of ab must contain 2 \cdot 3 So, 3 must be a factor of either a or b.
**A Preview of Lesson 9-2**

**Getting Started**

**Objective** Factor polynomials with algebra tiles.

**Materials**
- algebra tiles
- product mat

**Teach**

- Remind students that they used rectangles at the beginning of Lesson 9-1 to find factors of whole numbers. The procedure for finding the factors of polynomials with algebra tiles is very similar. The length and width of a modeled polynomial represent the factors of the polynomial.

**Assess**

- In Exercises 1–4, students need to recognize that they must arrange the tiles into a rectangle with a width greater than one in order to find the factors.
- In Exercises 5–9, students should recognize that the binomials that cannot be factored can only be modeled in a rectangle with a width of one.

**Activity 1** Use algebra tiles to factor $3x + 6$.

**Step 1** Model the polynomial $3x + 6$.

**Step 2** Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.

The rectangle has a width of 3 and a length of $x + 2$. So, $3x + 6 = 3(x + 2)$.

**Activity 2** Use algebra tiles to factor $x^2 - 4x$.

**Step 1** Model the polynomial $x^2 - 4x$.

**Step 2** Arrange the tiles into a rectangle.

The rectangle has a width of $x$ and a length of $x - 4$. So, $x^2 - 4x = x(x - 4)$.

**Model and Analyze**

Examples: $2x + 2$ can be factored and $2x + 1$ cannot be factored.

Use algebra tiles to factor each binomial.

1. $2x + 10$ 2. $2(x + 5)$ 3. $5x^2 + 2x$ 4. $3x - 8$ 5. $2(3x - 4)$ 6. $x(5x + 2)$ 7. $9 - 3x$ 8. $3(3 - x)$

Tell whether each binomial can be factored. Justify your answer with a drawing.

5. $4x - 10$ yes 6. $3x - 7$ no 7. $x^2 + 2x$ yes 8. $2x^2 + 3$ no


9. **MAKE A CONJECTURE** Write a paragraph that explains how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.

**Study Notebook**

You may wish to have students summarize this activity and what they learned from it.

**Resource Manager**

**Teaching Algebra with Manipulatives**
- pp. 10–11 (master for algebra tiles)
- p. 17 (master for product mat)
- p. 156 (student recording sheet)

**Glencoe Mathematics Classroom Manipulative Kit**
- algebra tiles
- product mat
Factoring Using the Distributive Property

What You’ll Learn
- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

Vocabulary
- factoring
- factoring by grouping

How can you determine how long a baseball will remain in the air?

Nolan Ryan, the greatest strike-out pitcher in the history of baseball, had a fastball clocked at 98 miles per hour or about 151 feet per second. If he threw a ball directly upward with the same velocity, the height $h$ of the ball in feet above the point at which he released it could be modeled by the formula $h = 151t - 16t^2$, where $t$ is the time in seconds. You can use factoring and the Zero Product Property to determine how long the ball would remain in the air before returning to his glove.

FACTOR BY USING THE DISTRIBUTIVE PROPERTY In Chapter 8, you used the Distributive Property to multiply a polynomial by a monomial.

$$2a(6a + 8) = 2a(6a) + 2a(8)$$
$$= 12a^2 + 16a$$

You can reverse this process to express a polynomial as the product of a monomial factor and a polynomial factor.

$$12a^2 + 16a = 2a(6a) + 2a(8)$$
$$= 2a(6a + 8)$$

Thus, a factored form of $12a^2 + 16a$ is $2a(6a + 8)$.

Factoring a polynomial means to find its completely factored form. The expression $2a(6a + 8)$ is not completely factored since $6a + 8$ can be factored as $2(3a + 4)$.

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $12a^2 + 16a$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$ Factor each number.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$ Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a$ or $4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$12a^2 + 16a = 4a(3a) + 4a(4)$ Rewrite each term using the GCF.

$= 4a(3a + 4)$ Simplify remaining factors.

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$. 

Lesson 9-2 Factoring Using the Distributive Property 481

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters
- Study Guide and Intervention, pp. 529–530
- Skills Practice, p. 531
- Practice, p. 532
- Reading to Learn Mathematics, p. 533
- Enrichment, p. 534
- Assessment, p. 573

Parent and Student Study Guide Workbook, p. 69
Prerequisite Skills Workbook, pp. 13–14
School-to-Career Masters, p. 17

Transparencies
5-Minute Check Transparency 9-2
Answer Key Transparencies

Technology
Interactive Chalkboard
2 Teach

FACTOR BY USING THE DISTRIBUTIVE PROPERTY

In-Class Examples

Teaching Tip Tell students that one way to find the remaining factors is to divide each term by the GCF.

1. Use the Distributive Property to factor each polynomial.
   a. $15x + 25x^2$ $5x(3 + 5x)$
   b. $12xy + 24xy^2 - 30x^2y^4$ $6xy(2 + 4y - 5xy^3)$

Teaching Tip Remind students that when using the FOIL method, they multiply the First terms, Outer terms, Inner terms, and Last terms.

2. Factor $2xy + 7x - 2y - 7$. $(x - 1)(2y + 7)$

3. Factor $15a - 3ab + 4b - 20$. $(-3a + 4)(b - 5)$

Concept Check

Factoring Using the Distributive Property Malcolm and Fatima each factored the polynomial $2ax + 6cx + ab + 3bc$. Malcolm’s answer was $(2x + b)(a + 3c)$ and Fatima’s was $(a + 3c)(2x + b)$. Which is correct? Explain your answer. Both are correct. The order in which factors are multiplied does not affect the product.

Study Tip

Factoring by Grouping
Sometimes you can group terms in more than one way when factoring a polynomial. For example, the polynomial in Example 2 could have been factored in the following way.

$4ab + 8b + 3a + 6$

$= (4ab + 3a) + (8b + 6)$

$= a(4b + 3) + 2(4b + 3)$

$= (4b + 3)(a + 2)$

Notice that this result is the same as in Example 2.

Study Tip

Factoring Trinomials
Since the order in which factors are multiplied does not affect the product, $(-5x + 3)(y - 7)$ is also a correct factoring of $-35x - 5xy + 3y - 21$.

Example 2 Use Grouping

Factor $4ab + 8b + 3a + 6$.

$4ab + 8b + 3a + 6$

$= (4ab + 8b) + (3a + 6)$ Group terms with common factors.

$= 4b(a + 2) + 3(a + 2)$ Factor the GCF from each grouping.

$= (a + 2)(4b + 3)$ Distributive Property

CHECK Use the FOIL method.

$(a + 2)(4b + 3) = (a)(4b) + (a)(3) + (2)(4b) + (2)(3)$

$= 4ab + 3a + 8b + 6$ $\checkmark$

Recognizing binomials that are additive inverses is often helpful when factoring by grouping. For example, $7 - y$ and $y - 7$ are additive inverses because their sum is 0. By rewriting $7 - y$ in the factored form $-1(y - 7)$, factoring by grouping is made possible in the following example.

Example 3 Use the Additive Inverse Property

Factor $35x - 5xy + 3y - 21$.

$35x - 5xy + 3y - 21 = (35x - 5xy) + (3y - 21)$ Group terms with common factors.

$= 5x(7 - y) + 3(y - 7)$ Factor the GCF from each grouping.

$= 5x(-1)(y - 7) + 3(y - 7)$ $7 - y = -1(y - 7)$

$= -5x(y - 7) + 3(y - 7)$ $5x(-1) = -5x$

$= (y - 7)(-5x + 3)$ Distributive Property

Concept Summary

Factoring by Grouping

- **Words**
  A polynomial can be factored by grouping if all of the following situations exist.
  - There are four or more terms.
  - Terms with common factors can be grouped together.
  - The two common factors are identical or are additive inverses of each other.

- **Symbols**
  $ax + bx + ay + by = x(a + b) + y(a + b)$
  $= (a + b)(x + y)$
SOLVE EQUATIONS BY FACTORING  Some equations can be solved by factoring. Consider the following products.

\begin{align*}
6(0) &= 0 & 0(-3) &= 0 & (5 - 5)(0) &= 0 & -2(-3 + 3) &= 0
\end{align*}

Notice that in each case, at least one of the factors is zero. These examples illustrate the Zero Product Property.

**Zero Product Property**

- **Words** If the product of two factors is 0, then at least one of the factors must be 0.
- **Symbols** For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b\) equal zero.

**Example 4** Solve an Equation in Factored Form

Solve \((d - 5)(3d + 4) = 0\). Then check the solutions.

If \((d - 5)(3d + 4) = 0\), then according to the Zero Product Property either \(d - 5 = 0\) or \(3d + 4 = 0\).

\[
\begin{align*}
(d - 5)(3d + 4) &= 0 & \text{Original equation} \\
(d - 5) &= 0 & \text{or} & & 3d + 4 &= 0 \\
(d - 5) &= 0 & \text{Set each factor equal to zero.} & & 3d &= -4 \\
& & & & d &= -\frac{4}{3} \\
\end{align*}
\]

The solution set is \(\left\{5, -\frac{4}{3}\right\}\).

**CHECK** Substitute 5 and \(-\frac{4}{3}\) for \(d\) in the original equation.

\[
\begin{align*}
(d - 5)(3d + 4) &= 0 & \left(d - 5\right)(3d + 4) &= 0 \\
(5 - 5)(3(5) + 4) &= 0 & \left(-\frac{4}{3} - 5\right)(3\left(-\frac{4}{3}\right) + 4) &= 0 \\
(0)(19) &= 0 & \left(-\frac{19}{3}\right)(0) &= 0 \\
0 &= 0 & 0 &= 0
\end{align*}
\]

If an equation can be written in the form \(ab = 0\), then the Zero Product Property can be applied to solve that equation.

**Example 5** Solve an Equation by Factoring

Solve \(x^2 = 7x\). Then check the solutions.

Write the equation so that it is of the form \(ab = 0\).

\[
\begin{align*}
x^2 &= 7x & \text{Original equation} \\
x^2 - 7x &= 0 & \text{Subtract 7x from each side.} \\
x(x - 7) &= 0 & \text{Factor the GCF of} \ x^2 \text{ and } -7x, \text{ which is } x. \\
x &= 0 \text{ or } x - 7 &= 0 & \text{Zero Product Property} \\
x &= 7 & \text{Solve each equation.}
\end{align*}
\]

The solution set is \(\{0, 7\}\). Check by substituting 0 and 7 for \(x\) in the original equation.
**Concept Check**

1. Write $4x^2 + 12x$ as a product of factors in three different ways. Then decide which of the three is the completely factored form. Explain your reasoning. **See margin.**

2. OPEN ENDED Give an example of the type of equation that can be solved by using the Zero Product Property.

3. Explain why $(x - 2)(x + 4) = 0$ cannot be solved by dividing each side by $x - 2$. The division would eliminate 2 as a solution.

Factor each polynomial. 7. $2ab(a^2b + 4 + 8ab^2)$

4. $9x^2 + 36x$ $9x(x + 4)$

5. $16xz - 40x^2z^2$ $8xz(2 - 5z)$

6. $24m^2n^2p + 36m^2n^2p$ $12m^2n^2(2p + 3n)$

7. $2a^3b^2 + 8ab + 16a^2b^3$ $2ab(2a^2 + 16ab^2 + 8)$

8. $5y^2 - 15y + 4y - 12$ $(5y - 4)(y - 3)$

9. $5c - 10d^2 + 2d - 4cd$ $(5c + 2d)(1 - 2c)$

Solve each equation. Check your solutions.

10. $h(h + 5) = 0$ $(0, -5)$

11. $(n - 4)(n + 2) = 0$ $(-2, 4)$

12. $5m = 3m^2$ $(0, 5, 3)$

**Application**

**PHYSICAL SCIENCE** For Exercises 13–15, use the information below and in the graphic.

A flare is launched from a life raft. The height $h$ of the flare in feet above the sea is modeled by the formula $h = 100t - 16t^2$, where $t$ is the time in seconds after the flare is launched.

13. At what height is the flare when it returns to the sea? **0 ft**

14. Let $h = 0$ in the equation $h = 100t - 16t^2$ and solve for $t$. **0, 6.25**

15. How many seconds will it take for the flare to return to the sea? Explain your reasoning. **6.25 s** The answer $0$ is not reasonable since it represents the time at which the flare is launched.

**Guided Practice**

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**Homework Help**

For Exercises 16–39, see p. 521A.

Factor each polynomial.

16. $5x + 30y$ $5(x + 6y)$

17. $16a + 4b$ $4(4a + b)$

18. $a^5b - a$ $a(ab^4 - 1)$

19. $x^3y^2 + x$ $x(xy^2 + 1)$

20. $21cd - 3d$ $3d(7c - 1)$

21. $14gh - 18h$ $2h(7g - 9)$

22. $15a^2y - 30ay$ $15ay(a - 2)$

23. $8bc^2 + 24hc$ $8hc(1 + 3c)$

24. $a + a^2b^2 + a^3b^3$ $a(1 + b^2 + ab^2)$

25. $18a^2bc^2 - 48abc^3$ $12abc(3a^2 - 4c)$

26. $3p^3q - 9pq^2 + 36pq$ $3pq(p^2 - 3q + 12)$

27. $a^2y^2 + 40xy^2z^2$ $40y^2z(a^2 + 25x)$

28. $12x^2y + 20x^3 + 32xy$ $4x(3x^2 + 5y + 8)$

29. $15x^2y + 20xy^2 + 16$ $5x(3x + 4y + 8)$

30. $x^2 + 5x + 7x + 35$ $(x^2 + 7x + 5) + 10$

31. $x^2 + 5x + 7x + 35$ $(x + 7)(x + 5)$

32. $4x^2 + 14x + 6x + 21$ $2x(x + 7) + 3(x + 7)$

33. $12y^2 + 9y + 8y + 6$ $3y(4y + 3) + 2(4y + 3)$

34. $6a^2 - 15a - 8a + 20$ $(2a + 5)(3a - 4)$

35. $18x^2 - 30x - 3x + 5$ $3(6x^2 - 10x - 1)$

36. $4ax + 3ay + 4bx + 3by$ $a(4x + 3y) + b(4x + 3y)$

37. $2my + 7x + 7m + 2xy$ $(m + 1)(2y + 7)$

38. $8ax - 6x - 12a + 9$ $(2a - 3)(4x - 6)$

39. $10x^2 - 14xy - 15x + 21y$ $(x - 3)(10x - 7y)$

**Extra Practice**

See page 840.

**GEOMETRY** For Exercises 40 and 41, use the following information. A quadrilateral has 4 sides and 2 diagonals. A pentagon has 5 sides and 5 diagonals. You can use $\frac{1}{2}n^2 - \frac{3}{2}n$ to find the number of diagonals in a polygon with $n$ sides.

40. Write this expression in factored form. $\frac{1}{2}n(n - 3)$

41. Find the number of diagonals in a decagon (10-sided polygon). **35**

**About the Exercises...**

**Organization by Objective**

- Factor by Using the Distributive Property: 16–39
- Solve Equations by Factoring: 48–59

**Odd/Even Assignments**

Exercises 16–39 and 44–59 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

**Basic**: 17–39 odd, 40–43, 47–59 odd, 62–81

**Average**: 17–39 odd, 42, 43, 45, 47–61 odd, 62–81

**Advanced**: 16–38 even, 44, 45, 46–60 even, 62–75 (optional: 76–81)

**All**: Practice Quiz 1 (1–10)

**Answers**

1. Sample answers: $4(x^2 + 3x)$, $x(4x + 12)$, or $4(x + 3)$; $4x(x + 3)$; $4x$ is the GCF of $4x^2$ and $12x$.

63. Answers should include the following.

- Let $h = 0$ in the equation $h = 151t - 16t^2$. To solve $0 = 151t - 16t^2$, factor the right-hand side as $(151 - 16t)$. Then, since $(151 - 16t) = 0$, either $t = 0$ or $151 - 16t = 0$. Solving each equation for $t$, we find that $t = 0$ or $t \approx 9.44$.

- The solution $t = 0$ represents the point at which the ball was initially thrown into the air. The solution $t \approx 9.44$ represents how long it took after the ball was thrown for it to return to the same height at which it was thrown.
**Factor by Using the Distributive Property** The Distributive Property has been used to multiply a polynomial by a monomial. It will also be used to express a polynomial in factored form. Compare the two columns in the table below.

<table>
<thead>
<tr>
<th>Monomial</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(x + 3)</td>
<td>2x + 6</td>
</tr>
<tr>
<td>3(y - 1)</td>
<td>3y - 3</td>
</tr>
<tr>
<td>(a - b)(a + b)</td>
<td>a^2 - b^2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor each polynomial:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 2(x + 4y)</td>
</tr>
<tr>
<td>2. 3y(x + 2z)</td>
</tr>
<tr>
<td>3. (a - b)(a + b)</td>
</tr>
</tbody>
</table>

**Check Using the FOIL method:**
- 3. (x + 1)(x - 2) |
- 4. (y + 3)(y - 3) |

**Factor each polynomial: |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. x^2 - 9</td>
</tr>
<tr>
<td>2. y^2 - 16</td>
</tr>
</tbody>
</table>

**Check using the difference of two squares:**
- 2. (x + 4)(x - 4) |

**Factor each polynomial: |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. x^2 - 9y^2</td>
</tr>
<tr>
<td>2. a^2 - b^2</td>
</tr>
</tbody>
</table>

**Check using the difference of two squares:**
- 2. (x + 3y)(x - 3y) |

**Factor each polynomial: |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (x + 2)(x - 2)</td>
</tr>
<tr>
<td>2. (y + 3)(y - 3)</td>
</tr>
</tbody>
</table>

**Check using the difference of two squares:**
- 2. (x + 4)(x - 4) |

**Example 1:**
1. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

2. **Solve each equation. Check your solutions.**
   - **Example:** 2x - 3 = 7
     - **Solution:**
       - 2x = 10
       - x = 5

3. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

4. **Factor each polynomial:**
   - **Solution:**
     - x^2 - 4
     - (x + 2)(x - 2)

5. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

6. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

7. **Factor each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

8. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

9. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

10. **Factor each polynomial:**
    - **Solution:**
      - x^2 - 4
      - (x + 2)(x - 2)

11. **Check using the difference of two squares:**
    - **Solution:**
      - (x + 2)(x - 2)

**Example 2:**
1. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

2. **Solve each equation. Check your solutions.**
   - **Example:** 2x - 3 = 7
     - **Solution:**
       - 2x = 10
       - x = 5

3. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

4. **Factor each polynomial:**
   - **Solution:**
     - x^2 - 4
     - (x + 2)(x - 2)

5. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

6. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

7. **Factor each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

8. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

9. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

10. **Factor each polynomial:**
    - **Solution:**
      - x^2 - 4
      - (x + 2)(x - 2)

11. **Check using the difference of two squares:**
    - **Solution:**
      - (x + 2)(x - 2)

**Example 3:**
1. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

2. **Solve each equation. Check your solutions.**
   - **Example:** 2x - 3 = 7
     - **Solution:**
       - 2x = 10
       - x = 5

3. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

4. **Factor each polynomial:**
   - **Solution:**
     - x^2 - 4
     - (x + 2)(x - 2)

5. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

6. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

7. **Factor each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

8. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

9. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

10. **Factor each polynomial:**
    - **Solution:**
      - x^2 - 4
      - (x + 2)(x - 2)

11. **Check using the difference of two squares:**
    - **Solution:**
      - (x + 2)(x - 2)

**Example 4:**
1. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

2. **Solve each equation. Check your solutions.**
   - **Example:** 2x - 3 = 7
     - **Solution:**
       - 2x = 10
       - x = 5

3. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

4. **Factor each polynomial:**
   - **Solution:**
     - x^2 - 4
     - (x + 2)(x - 2)

5. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

6. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

7. **Factor each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

8. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

9. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

10. **Factor each polynomial:**
    - **Solution:**
      - x^2 - 4
      - (x + 2)(x - 2)

11. **Check using the difference of two squares:**
    - **Solution:**
      - (x + 2)(x - 2)

**Example 5:**
1. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

2. **Solve each equation. Check your solutions.**
   - **Example:** 2x - 3 = 7
     - **Solution:**
       - 2x = 10
       - x = 5

3. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

4. **Factor each polynomial:**
   - **Solution:**
     - x^2 - 4
     - (x + 2)(x - 2)

5. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

6. **Find an expression for the area of a square with the given perimeter.**
   - **Solution:**
     - **Perimeter** = 4s
     - **Area** = s^2

7. **Factor each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

8. **Check using the difference of two squares:**
   - **Solution:**
     - (x + 2)(x - 2)

9. **Write each polynomial:**
   - **Solution:**
     - 2x^2 - 3x + 1
     - (2x - 1)(x - 1)

10. **Factor each polynomial:**
    - **Solution:**
      - x^2 - 4
      - (x + 2)(x - 2)

11. **Check using the difference of two squares:**
    - **Solution:**
      - (x + 2)(x - 2)
Open-Ended Assessment

Speaking  Ask a volunteer to describe the similarities and differences between factoring using grouping, and factoring using the additive inverse property. Encourage other students to ask questions.

Getting Ready for Lesson 9-3

PREREQUISITE SKILL  Students will learn to factor trinomials in Lesson 9-3. It is important that students recall how to multiply polynomials to check that they correctly factored trinomials. Use Exercises 76–81 to determine your students’ familiarity with multiplying polynomials.

Assessment Options

Practice Quiz 1  The quiz provides students with a brief review of the concepts and skills in Lessons 9-1 and 9-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 9-1 and 9-2) is available on p. 573 of the Chapter 9 Resource Masters.

Answer

1. 1, 3, 5, 9, 15, 25, 45, 75, 225; composite

486 Chapter 9  Factoring
Factoring Trinomials

You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle’s length and width are factors of the area.

**Activity 1** Use algebra tiles to factor $x^2 + 6x + 5$.

**Step 1** Model the polynomial $x^2 + 6x + 5$.

**Step 2** Place the $x^2$ tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Because 5 is prime, the 5 tiles can be arranged in a rectangle in one way, a 1-by-5 rectangle.

**Step 3** Complete the rectangle with the $x$ tiles.

The rectangle has a width of $x + 1$ and a length of $x + 5$. Therefore, $x^2 + 6x + 5 = (x + 1)(x + 5)$.

**Activity 2** Use algebra tiles to factor $x^2 + 7x + 6$.

**Step 1** Model the polynomial $x^2 + 7x + 6$.

**Step 2** Place the $x^2$ tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Since 6 = $2 \times 3$, try a 2-by-3 rectangle. Try to complete the rectangle. Notice that there are two extra $x$ tiles.

(continued on the next page)
**Step 3** Arrange the 1 tiles into a 1-by-6 rectangular array. This time you can complete the rectangle with the \( x \) tiles.

The rectangle has a width of \( x + 1 \) and a length of \( x + 6 \). Therefore,

\[
x^2 + 7x + 6 = (x + 1)(x + 6).
\]

**Activity 3** Use algebra tiles to factor \( x^2 - 2x - 3 \).

**Step 1** Model the polynomial \( x^2 - 2x - 3 \).

**Step 2** Place the \( x^2 \) tile at the corner of the product mat. Arrange the 1 tiles into a 1-by-3 rectangular array as shown.

**Step 3** Place the \( x \) tile as shown. Recall that you can add zero-pairs without changing the value of the polynomial. In this case, add a zero pair of \( x \) tiles.

The rectangle has a width of \( x + 1 \) and a length of \( x - 3 \). Therefore,

\[
x^2 - 2x - 3 = (x + 1)(x - 3).
\]

**Model**

1. \( (x + 3)(x + 1) \)  
2. \( (x + 4)(x + 1) \)  
3. \( (x - 3)(x + 2) \)  
4. \( (x - 2)(x - 1) \)

Use algebra tiles to factor each trinomial.

1. \( x^2 + 4x + 3 \)  
2. \( x^2 + 5x + 4 \)  
3. \( x^2 - x - 6 \)  
4. \( x^2 - 3x + 2 \)
5. \( x^2 + 7x + 12 \)  
6. \( x^2 - 4x + 4 \)  
7. \( x^2 - x - 2 \)  
8. \( x^2 - 6x + 8 \)

\( (x + 4)(x + 3) \) \( (x - 2)(x - 2) \) \( (x + 1)(x - 2) \) \( (x - 4)(x - 2) \)
Factoring Trinomials: 
\[ x^2 + bx + c \]

**What You’ll Learn**
- Factor trinomials of the form \( x^2 + bx + c \).
- Solve equations of the form \( x^2 + bx + c = 0 \).

**How can factoring be used to find the dimensions of a garden?**

Tamika has enough bricks to make a 30-foot border around the rectangular vegetable garden she is planting. The booklet she got from the nursery says that the plants will need a space of 54 square feet to grow. What should the dimensions of her garden be? To solve this problem, you need to find two numbers whose product is 54 and whose sum is 15, half the perimeter of the garden.

**FOIL**

In Lesson 9-1, you learned that when two numbers are multiplied, each number is a factor of the product. Similarly, when two binomials are multiplied, each binomial is a factor of the product.

To factor some trinomials, you will use the pattern for multiplying two binomials. Study the following example.

\[
(x + 2)(x + 3) = (x \cdot x) + (x \cdot 3) + (2 \cdot 2) + (2 \cdot 3)
\]

\[
= x^2 + 3x + 2x + 6
\]

\[
= x^2 + 5x + 6
\]

Observe the following pattern in this multiplication.

\[
(x + 2)(x + 3) = x^2 + (3 + 2)x + 6
\]

\[
= x^2 + 5x + 6
\]

Notice that the coefficient of the middle term is the sum of \( m \) and \( n \) and the last term is the product of \( m \) and \( n \). This pattern can be used to factor quadratic trinomials of the form \( x^2 + bx + c \).

**Key Concept**

**Factoring \( x^2 + bx + c \)**

- **Words**
  To factor quadratic trinomials of the form \( x^2 + bx + c \), find two integers, \( m \) and \( n \), whose sum is equal to \( b \) and whose product is equal to \( c \). Then write \( x^2 + bx + c \) using the pattern \((x + m)(x + n)\).

- **Symbols**
  \[ x^2 + bx + c = (x + m)(x + n) \text{ when } m + n = b \text{ and } mn = c. \]

- **Example**
  \[ x^2 + 5x + 6 = (x + 2)(x + 3), \text{ since } 2 + 3 = 5 \text{ and } 2 \cdot 3 = 6. \]

**Workbook and Reproducible Masters**

- **Chapter 9 Resource Masters**
  - Study Guide and Intervention, pp. 535–536
  - Skills Practice, p. 537
  - Practice, p. 538
  - Reading to Learn Mathematics, p. 539
  - Enrichment, p. 540
  - Assessment, pp. 573, 575

- **Parent and Student Study Guide Workbook**, p. 70

**Resource Manager**

- **Transparencies**
  - 5-Minute Check Transparency 9-3
  - Answer Key Transparencies

- **Technology**
  - AlgePASS: Tutorial Plus, Lessons 24, 25
  - Interactive Chalkboard
The concept of factoring trinomials as introduced in this lesson may seem somewhat abstract to some students. Whenever you introduce abstract concepts, it is good to reinforce them with a concrete example. After introducing factoring trinomials, refer students back to the lesson opener problem. Ask students to describe any similarities they notice between finding the dimensions of the garden and factoring a trinomial.

In-Class Examples

**Teaching Tip** Tell students that the order in which they record the factors does not matter. So, \((x + 4)(x + 2)\) is also correct.

1. Factor \(x^2 + 7x + 12\).
   \((x + 3)(x + 4)\)

**Teaching Tip** If students use the graphing calculator to check their factoring, make sure they clear all other functions from the Y= list, and clear all other drawings from the draw menu.

2. Factor \(x^2 - 12x + 27\).
   \((x - 3)(x - 9)\)

---

**Study Tip**

**Testing Factors**

Once you find the correct factors, there is no need to test any other factors. Therefore, it is not necessary to test \(-4\) and \(-4\) in Example 2.

---

**TEACHING TIP**

Caution students that two graphs may appear to coincide in the standard viewing window, but they do not. Have them use the TABLE feature to verify the identical \(y\) values.

---

### Example 1 \(b\) and \(c\) Are Positive

Factor \(x^2 + 6x + 8\).

In this trinomial, \(b = 6\) and \(c = 8\). You need to find two numbers whose sum is 6 and whose product is 8. Make an organized list of the factors of 8, and look for the pair of factors whose sum is 6.

<table>
<thead>
<tr>
<th>Factors of 8</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 8</td>
<td>9</td>
</tr>
<tr>
<td>2, 4</td>
<td>6</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 4.

\[x^2 + 6x + 8 = (x + m)(x + n)\]

Write the pattern.

\[=(x + 2)(x + 4)\]

\[m = 2\] and \(n = 4\)

**CHECK**

You can check this result by multiplying the two factors.

\[F O I L\]

\[= x^2 + 6x + 8\]

Simplify.

---

When factoring a trinomial where \(b\) is negative and \(c\) is positive, you can use what you know about the product of binomials to help narrow the list of possible factors.

### Example 2 \(b\) is Negative and \(c\) is Positive

Factor \(x^2 - 10x + 16\).

In this trinomial, \(b = -10\) and \(c = 16\). This means that \(m + n\) is negative and \(mn\) is positive. So \(m\) and \(n\) must both be negative. Therefore, make a list of the negative factors of 16, and look for the pair of factors whose sum is \(-10\).

<table>
<thead>
<tr>
<th>Factors of 16</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, -16)</td>
<td>(-17)</td>
</tr>
<tr>
<td>(-2, -8)</td>
<td>(-10)</td>
</tr>
<tr>
<td>(-4, -4)</td>
<td>(-8)</td>
</tr>
</tbody>
</table>

The correct factors are \(-2\) and \(-8\).

\[x^2 - 10x + 16 = (x + m)(x + n)\]

Write the pattern.

\[=(x - 2)(x - 8)\]

\[m = -2\] and \(n = -8\)

**CHECK**

You can check this result by using a graphing calculator. Graph \(y = x^2 - 10x + 16\) and \(y = (x - 2)(x - 8)\) on the same screen.

Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly.

You will find that keeping an organized list of the factors you have tested is particularly important when factoring a trinomial like \(x^2 + x - 12\), where the value of \(c\) is negative.

---

**Differentiated Instruction**

**Kinesthetic** As students are learning the rules for factoring trinomials, encourage them to use algebra tiles to confirm their results. Students should soon realize that the greater the values of \(b\) and \(c\) in the trinomials, the more cumbersome algebra tiles become, which should reinforce the importance of learning to factor using the method in the text.
Example 3  b Is Positive and c Is Negative

Factor $x^2 + x - 12$.
In this trinomial, $b = 1$ and $c = -12$. This means that $m + n$ is positive and $mn$ is negative. So either $m$ or $n$ is negative, but not both. Therefore, make a list of the factors of $-12$, where one factor of each pair is negative. Look for the pair of factors whose sum is 1.

<table>
<thead>
<tr>
<th>Factors of $-12$</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, -12</td>
<td>-11</td>
</tr>
<tr>
<td>-1, 12</td>
<td>11</td>
</tr>
<tr>
<td>2, -6</td>
<td>-4</td>
</tr>
<tr>
<td>-2, 6</td>
<td>4</td>
</tr>
<tr>
<td>3, -4</td>
<td>-1</td>
</tr>
<tr>
<td>-3, 4</td>
<td>1</td>
</tr>
</tbody>
</table>

The correct factors are $-3$ and $-4$.

$x^2 + x - 12 = (x + m)(x + n)$ Write the pattern.

$= (x - 3)(x - 4)$ $m = -3$ and $n = 4$

Example 4  b Is Negative and c Is Negative

Factor $x^2 - 7x - 18$.
Since $b = -7$ and $c = -18$, $m + n$ is negative and $mn$ is negative. So either $m$ or $n$ is negative, but not both.

<table>
<thead>
<tr>
<th>Factors of $-18$</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, -18</td>
<td>-17</td>
</tr>
<tr>
<td>-1, 18</td>
<td>17</td>
</tr>
<tr>
<td>2, -9</td>
<td>-7</td>
</tr>
</tbody>
</table>

The correct factors are $2$ and $-9$.

$x^2 - 7x - 18 = (x + m)(x + n)$ Write the pattern.

$= (x + 2)(x - 9)$ $m = 2$ and $n = -9$

SOLVE EQUATIONS BY FACTORING Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 5  Solve an Equation by Factoring

Solve $x^2 + 5x = 6$. Check your solutions.

$x^2 + 5x = 6$ Original equation
$x^2 + 5x - 6 = 0$ Rewrite the equation so that one side equals 0.
$(x - 1)(x + 6) = 0$ Factor.

$x - 1 = 0$ or $x + 6 = 0$ Zero Product Property
$x = 1$ $x = -6$ Solve each equation.

The solution set is $\{1, -6\}$.

CHECK Substitute 1 and $-6$ for $x$ in the original equation.

$x^2 + 5x = 6$
$(1)^2 + 5(1) = 6$
$1 + 5 = 6$ True

$x^2 + 5x = 6$
$(-6)^2 + 5(-6) = 6$
$36 - 30 = 6$ True

www.algebra1.com/extra_examples
Solving a trinomial equation.

1. In this trinomial, 
   
   \[ x^2 - 14x + 40 = 0 \]
   
   Sample answer:  
   
   \( \{4, 10\} \)
   
   **Odd/Even Assignments**
   
   Exercises 17–53 are structured so that students practice the same concepts whether they are assigned odd or even problems.

   **Alert!** Exercises 66–69 require a graphing calculator.

### Check for Understanding

**Concept Check**

1. Explain why, when factoring \( x^2 + 6x + 9 \), it is not necessary to check the sum of the factor pairs \(-1\) and \(-9\) or \(-3\) and \(-3\). *See margin.*

2. **OPEN ENDED** Give an example of an equation that can be solved using the factoring techniques presented in this lesson. Then, solve your equation.

3. **FIND THE ERROR** Peter and Aleta are solving \( x^2 + 2x = 15 \).

<table>
<thead>
<tr>
<th>Peter</th>
<th>Aleta</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + 2x = 15 )</td>
<td>( x^2 + 2x = 15 )</td>
</tr>
<tr>
<td>( x + 5 )</td>
<td>( x + 5 )</td>
</tr>
<tr>
<td>( x + 15 )</td>
<td>( x = 3 )</td>
</tr>
<tr>
<td>(15)</td>
<td>( x = -5 )</td>
</tr>
</tbody>
</table>

   Who is correct? Explain your reasoning. Aleta; to use the Zero Product Property, one side of the equation must equal zero.

### Guided Practice

Factor each trinomial.

4. \( (x + 3)(x + 8) \)
5. \( (c - 1)(c - 2) \)
6. \( (n - 3)(n + 16) \)
7. \( (p + 5)(p - 7) \)
8. \( (a + 3)(a + 24) \)
9. \( (x - 3y)(x + y) \)

### Answers

1. In this trinomial, \( b = 6 \) and \( c = 9 \). This means that \( m + n \) is positive and \( mn \) is positive. Only two positive numbers have both a positive sum and product. Therefore, negative factors of 9 need not be considered.

2. **Answers**

   - 10. \( \{-1, -6\} \)
   - 11. \( \{-9, 4\} \)
   - 12. \( \{-2, 21\} \)
   - 13. \( \{-9, -1\} \)
   - 14. \( \{-11, 2\} \)
   - 15. \( \{-7, 10\} \)
   - 16. \( \{a + 3\}(a + 5) \)
   - 17. \( \{x + 3\}(x + 9) \)
   - 18. \( \{c + 5\}(c + 7) \)
   - 19. \( \{y + 10\}(y + 3) \)
   - 20. \( \{y + 10\}(y + 3) \)
   - 21. \( \{m - 1\}(m - 21) \)
   - 22. \( \{m - 1\}(m - 21) \)
   - 23. \( \{p - 8\}(p - 9) \)
   - 24. \( \{p - 8\}(p - 9) \)
   - 25. \( \{x - 1\}(x + 7) \)
   - 26. \( \{b - 4\}(b + 5) \)
   - 27. \( \{h - 5\}(h + 8) \)
   - 28. \( \{n - 6\}(n + 9) \)
   - 29. \( \{y - 7\}(y + 6) \)
   - 30. \( \{z + 2\}(z - 20) \)
   - 31. \( \{w + 12\}(w - 6) \)
   - 32. \( \{x - 2\}(x + 15) \)
   - 33. \( \{a - b\}(a + 4b) \)
   - 34. \( \{x - 4\}(x - 9y) \)
Solve each equation. Check your solutions. \(10–15.\) See margin.

10. \(n^2 + 7n + 6 = 0\) 
11. \(a^2 + 5a - 36 = 0\) 
12. \(p^2 - 19p = 42 = 0\)
13. \(y^2 + 9 = -10y\) 
14. \(9x + x^2 = 22\) 
15. \(b^2 - 3d = 70\)

Application 16. NUMBER THEORY Find two consecutive integers whose product is 156.

For Exercises 61 and 62, use the following information.

61. Solve each equation. Check your solutions.

62. Write an expression for the area of the field. \([w(w + 52)]^2\) m²

63. Write an expression for the area of the Regimental image. \(10 - 10x - 5 = 15, or a = 22x + 105 = 0\)

64. Find the dimensions of the Regimental image. 39 by 5 in.

Lesson 9-3 Factoring Trinomials: \(x^2 + bx + c\) p. 493

Enrichment, p. 540

Puzzling Primes

A prime number is a two-factor, 1 and the number itself. The number 1 is not a prime number, if it is a prime number, it is not a prime number, if it is a prime number, it is not a prime number.

Enrichment, p. 540

Puzzling Primes

A prime number is a two-factor, 1 and the number itself. The number 1 is not a prime number, if it is a prime number, it is not a prime number, if it is a prime number, it is not a prime number.

Enrichment, p. 540

Puzzling Primes

A prime number is a two-factor, 1 and the number itself. The number 1 is not a prime number, if it is a prime number, it is not a prime number, if it is a prime number, it is not a prime number.

Enrichment, p. 540

Puzzling Primes

A prime number is a two-factor, 1 and the number itself. The number 1 is not a prime number, if it is a prime number, it is not a prime number, if it is a prime number, it is not a prime number.
Open-Ended Assessment

Speaking  Ask volunteers to brainstorm a mnemonic device that will help them remember how to factor trinomials with different positive and negative values of \( b \) and \( c \). Then write example trinomials on the chalkboard and have students factor them using the mnemonic devices as a guide.

Getting Ready for Lesson 9-4

PREREQUISITE SKILL  Students will learn to factor additional types of trinomials in Lesson 9-4 using factoring by grouping. Use Exercises 78–83 to determine if the students’ familiarity with factoring by grouping.

Assessment Options

Quiz (Lesson 9-3) is available on p. 573 of the Chapter 9 Resource Masters.

Mid-Chapter Test (Lessons 9-1 through 9-3) is available on p. 575 of the Chapter 9 Resource Masters.

Answer

63. Answers should include the following.
   • You would use a guess-and-check process, checking which pairs added to 15.
   • To factor a trinomial of the form \( x^2 + ax + c \), you also use a guess-and-check process, list the factors of \( c \), and check to see which ones add to \( a \).

64. Which is the factored form of \( x^2 - 17x + 42? \) C

   \[ (x - 1)(y - 42) \] (x - 2)(x - 21)
   \[ (x - 3)(x - 14) \] (x - 6)(x - 7)

65. GRID IN  What is the positive solution of \( p^2 - 13p - 30 = 0? \) 15

Maintain Your Skills

Mixed Review

Solve each equation. Check your solutions.  

70. \( (x + 3)(2x - 5) = 0 \)  
71. \( b(7b - 4) = 0 \) \( (0, \frac{4}{7}) \)  
72. \( 5y^2 = -9y \) \( \left\{ -\frac{9}{5}, 0 \right\} \)

Find the GCF of each set of monomials.  

73. 24, 36, 72  
74. \( 9p^4q^2, 21p^3q^3 \) \( 3p^2q^2 \)  
75. \( 30x^2y^5, 20x^2y^7, 75x^3y^4 \) \( 5x^2y^4 \)

INTERNET  For Exercises 76 and 77, use the graph at the right.  

76. Find the percent increase in the number of domain registrations from 1997 to 2000. \( 1731\% \)  
77. Use your answer from Exercise 76 to verify the claim that registrations grew more than 18-fold from 1997 to 2000 is correct. \( (1.54) + 17.31(1.54) = (1 + 17.31)(1.54) \) or \( 18.31(1.54) \)

78. \( (y + 3)(3y + 2) \)  
79. \( (a + 4)(3a + 2) \)  
80. \( (x + 2)(4x + 3) \)

Getting Ready for the Next Lesson

PREREQUISITE SKILL  Factor each polynomial.  

78. \( 3y^2 + 2y + 9y + 6 \)  
79. \( 3a^2 + 2a + 12a + 8 \)  
80. \( 4x^2 + 3x + 8x + 6 \)  
81. \( 2p^2 + 6p + 7p - 21 \)  
82. \( 3b^2 + 7b - 12b - 28 \)  
83. \( 4y^2 - 2w - 6y + 3 \)  
\( (2p + 7)(p - 3) \)  
\( (b - 4)(3b + 7) \)  
\( (2g - 3)(2g - 1) \)

Online Lesson Plans

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. Experience TODAY, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
Factoring Trinomials: $ax^2 + bx + c$

**What You’ll Learn**
- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

**Study Tip**
**Look Back**
To review factoring by grouping, see Lesson 9-2.

**Vocabulary**
- prime polynomial

**How can algebra tiles be used to factor $2x^2 + 7x + 6$?**

The factors of $2x^2 + 7x + 6$ are the dimensions of the rectangle formed by the algebra tiles shown below.

![Algebra tiles](image)

The process you use to form the rectangle is the same mental process you can use to factor this trinomial algebraically.

**FACTOR $ax^2 + bx + c$**

For trinomials of the form $x^2 + bx + c$, the coefficient of $x^2$ is 1. To factor trinomials of this form, you find the factors of $c$ whose sum is $b$.

We can modify this approach to factor trinomials whose leading coefficient is not 1.

$$\begin{align*}
(2x + 5)(3x + 1) &= 6x^2 + 2x + 15x + 5 \\
\text{Use the FOIL method.} \\
2 \cdot 15 &= 30 \\
6 \cdot 5 &= 30
\end{align*}$$

Observe the following pattern in this product.

$$\begin{align*}
6x^2 + 2x + 15x + 5 &\quad ax^2 + mx + nx + c \\
6x^2 + 17x + 5 &\quad ax^2 + bx + c \\
2 + 15 &= 17 \text{ and } 2 \cdot 15 = 6 \cdot 5 &\quad m + n = b \text{ and } mn = ac
\end{align*}$$

You can use this pattern and the method of factoring by grouping to factor $6x^2 + 17x + 5$. Find two numbers, $m$ and $n$, whose product is $6 \cdot 5$ or 30 and whose sum is 17.

<table>
<thead>
<tr>
<th>Factors of 30</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 30</td>
<td>31</td>
</tr>
<tr>
<td>2, 15</td>
<td>17</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 15.

$$\begin{align*}
6x^2 + 17x + 5 &= 6x^2 + mx + nx + 5 \\
&= 6x^2 + 2x + 15x + 5 \\
&= (6x^2 + 2x) + (15x + 5) \\
&= 2x(3x + 1) + 5(3x + 1) \\
&= (3x + 1)(2x + 5)
\end{align*}$$

Therefore, $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$.

**Lesson 9-4 Factoring Trinomials: $ax^2 + bx + c$**

**5-Minute Check Transparency 9-4**
Use as a quiz or review of Lesson 9-3.

**Mathematical Background**
Notes are available for this lesson on p. 472D.

**How can algebra tiles be used to factor $2x^2 + 7x + 6$?**

**Ask students:**
- When you form a rectangle with algebra tiles to model a trinomial such as the one given, how do you know the factors? The length of the rectangle is one factor, and the height is the other factor.
- How is the trinomial $2x^2 + 7x + 6$ different from those that you learned how to factor in Lesson 9-3? The $x^2$ term is multiplied by a constant (2).
- What would the rectangle formed by the given algebra tiles look like? What are the factors of the trinomial?

$$\begin{array}{|c|c|c|}
\hline
x^2 & x^2 & x \\
\hline
x & x & 1 \\
\hline
\end{array}$$

$$\begin{align*}
(2x + 3)(x + 2)
\end{align*}$$

**Resource Manager**

**Workbook and Reproducible Masters**
- **Chapter 9 Resource Masters**
  - Study Guide and Intervention, pp. 541–542
  - Skills Practice, p. 543
  - Practice, p. 544
  - Reading to Learn Mathematics, p. 545
  - Enrichment, p. 546
- **Graphing Calculator and Spreadsheet Masters**, p. 39
- **Parent and Student Study Guide Workbook**, p. 71
- **Prerequisite Skills Workbook**, pp. 13–14

**Transparencies**
- 5-Minute Check Transparency 9-4
- Answer Key Transparencies

**Technology**
- AlgePASS: Tutorial Plus, Lesson 26
- Interactive Chalkboard
FACTOR $ax^2 + bx + c$

**In-Class Examples**

**Teaching Tip** In Example 1b of the Student Edition, $mn$ is a large number, which has quite a few factors. Have students start listing the factors that can easily be determined by mental math. More than likely, the factors that equal $m + n$ can be found this way, without too much calculation.

1. **Factor** $5x^2 + 27x + 10$.
   
   $$(5x + 2)(x + 5)$$

2. **Factor** $24x^2 - 22x + 3$.
   
   $$(4x - 3)(6x - 1)$$

**Teaching Tip** Remind students not to forget to record the common factor that they factored out of the trinomial when they record the other two factors.

2. **Factor** $4x^2 + 24x + 32$.
   
   $$4(x + 2)(x + 4)$$

**Example 1** Factor $ax^2 + bx + c$

**a. Factor** $7x^2 + 22x + 3$.

In this trinomial, $a = 7$, $b = 22$ and $c = 3$. You need to find two numbers whose sum is 22 and whose product is $7 \cdot 3$ or 21. Make an organized list of the factors of 21 and look for the pair of factors whose sum is 22.

<table>
<thead>
<tr>
<th>Factors of 21</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 21</td>
<td>22</td>
</tr>
</tbody>
</table>

The correct factors are 1 and 21.

$$7x^2 + 22x + 3 = 7x^2 + mx + nx + 3$$  
Write the pattern.

$$= 7x^2 + 1x + 21x + 3$$  
$m = 1$ and $n = 21$

$$= (7x^2 + 1x) + (21x + 3)$$  
Group terms with common factors.

$$= x(7x + 1) + 3(7x + 1)$$  
Factor the GCF from each grouping.

$$= (7x + 1)(x + 3)$$  
Distributive Property

**CHECK** You can check this result by multiplying the two factors.

$$F \cdot O \cdot I \cdot L$$

$$(7x + 1)(x + 3) = 7x^2 + 21x + x + 3$$  
FOIL method

$$= 7x^2 + 22x + 3$$  
Simplify.

**b. Factor** $10x^2 - 43x + 28$.

In this trinomial, $a = 10$, $b = -43$ and $c = 28$. Since $b$ is negative, $m + n$ is negative. Since $c$ is positive, $mn$ is positive. So $m$ and $n$ must both be negative. Therefore, make a list of the negative factors of $10 \cdot 28$ or 280, and look for the pair of factors whose sum is $-43$.

<table>
<thead>
<tr>
<th>Factors of 280</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$, $-280$</td>
<td>$-281$</td>
</tr>
<tr>
<td>$-2$, $-140$</td>
<td>$-142$</td>
</tr>
<tr>
<td>$-4$, $-70$</td>
<td>$-74$</td>
</tr>
<tr>
<td>$-5$, $-56$</td>
<td>$-61$</td>
</tr>
<tr>
<td>$-7$, $-40$</td>
<td>$-47$</td>
</tr>
<tr>
<td>$-8$, $-35$</td>
<td>$-43$</td>
</tr>
</tbody>
</table>

The correct factors are $-8$ and $-35$.

$$10x^2 - 43x + 28$$

$$= 10x^2 + mx + nx + 28$$  
Write the pattern.

$$= 10x^2 + (-8)x + (-35)x + 28$$  
$m = -8$ and $n = -35$

$$= (10x^2 - 8x) + (-35x + 28)$$  
Group terms with common factors.

$$= 2x(5x - 4) + 7(-5x + 4)$$  
Factor the GCF from each grouping.

$$= 2x(5x - 4) + 7(-1)(5x - 4)$$  
Factor the GCF from each grouping.

$$= 2x(5x - 4) + (-7)(5x - 4)$$  
Simplify.

$$= (5x - 4)(2x - 7)$$  
Distributive Property

Sometimes the terms of a trinomial will contain a common factor. In these cases, first use the Distributive Property to factor out the common factor. Then factor the trinomial.

**Example 2** Factor When $a$, $b$, and $c$ Have a Common Factor

Factor $3x^2 + 24x + 45$.

Notice that the GCF of the terms $3x^2$, $24x$, and $45$ is 3. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

$$3x^2 + 24x + 45 = 3(x^2 + 8x + 15)$$  
Distributive Property

**Differentiated Instruction**

**Interpersonal** Place students in groups to factor polynomials such as those in Examples 1 and 2. Have each group member find one or two factors for $mn$, depending on the number of factors and number of students in the group. By dividing the labor, students should be able to quickly find the factors for $mn$ that sum to $m + n$. Once they find the factors, have students complete the factoring as a group.
Now factor $x^2 + 8x + 15$. Since the lead coefficient is 1, find two factors of 15 whose sum is 8.

### Factors of 15 | Sum of Factors
--- | ---
1, 15 | 16
3, 5 | 8

The correct factors are 2 and 15.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Thus, the complete factorization of $3x^2 + 24x + 45$ is $3(x + 3)(x + 5)$.

A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**.

### Example 3 Determine Whether a Polynomial Is Prime

Factor $2x^2 + 5x - 2$.

In this trinomial, $a = 2$, $b = 5$ and $c = -2$. Since $b$ is positive, $m + n$ is positive. Since $c$ is negative, $mn$ is negative. So either $m$ or $n$ is negative, but not both. Therefore, make a list of the factors of $2 \cdot -2$ or $-4$, where one factor in each pair is negative. Look for a pair of factors whose sum is 5.

### Factors of $-4$ | Sum of Factors
--- | ---
1, $-4$ | $-3$
$-1$, 4 | 3
2, $-2$ | 0

There are no factors whose sum is 5. Therefore, $2x^2 + 5x - 2$ cannot be factored using integers. Thus, $2x^2 + 5x - 2$ is a prime polynomial.

### SOLVE EQUATIONS BY FACTORING

Some equations of the form $ax^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

### Example 4 Solve Equations by Factoring

**Solve** $8x^2 - 9a - 5 = 4 - 3a$.

**Check** each solution by substituting it into the original equation.

**CHECK**

$$8\left(\frac{-3}{4}\right)^2 - 9\left(\frac{-3}{4}\right) - 5 = 4 - 3\left(\frac{-3}{4}\right)$$

$$8\left(\frac{9}{16}\right) - 9\left(\frac{-3}{4}\right) - 5 = 4 - 3\left(\frac{-3}{4}\right)$$

$$\frac{9}{2} + 27 + 5 \neq 4 + \frac{9}{4}$$

$$\frac{25}{4} \neq \frac{25}{4}$$

**3.5 seconds**

### In-Class Example

**Teaching Tip** Make sure students list all possible factors of $mn$, including both positive and negative factors, before they decide the polynomial is prime.

**Factor** $3x^2 + 7x - 5$.

**Model Rockets** Ms. Nguyen’s science class built an air-launched model rocket for a competition. When they test-launched their rocket outside the classroom, the rocket landed in a nearby tree. If the launch pad was 2 feet above the ground, the initial velocity of the rocket was 64 feet per second, and the rocket landed 30 feet above the ground, how long was the rocket in flight? Use the equation $h = -16t^2 + vt + s$. Please calculate the answer.
### Example 5 Solve Real-World Problems by Factoring

#### PEP RALLY

At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student in the stands catches it on its way down 26 feet above the gym floor?

Use the model for vertical motion.

1. **Vertical motion model**
   
   
   
   
   \[ h = -16t^2 + vt + s \]

2. **Subtract 26 from each side.**
   
   
   
   
   \[ h - 26 = -16t^2 + vt + s - 26 \]

3. **Factor out -2.**
   
   
   
   
   \[ -2(8t^2 - 8t + 10) = 0 \]

4. **Divide each side by -2.**
   
   
   
   
   \[ 8t^2 - 8t + 10 = 0 \]

5. **Factor 8t^2 - 8t + 10.**
   
   
   
   
   \[ 8t^2 - 8t + 10 = (8t - 5)(t - 2) \]

6. **Zero Product Property**
   
   
   
   
   \[ 8t - 5 = 0 \text{ or } t - 2 = 0 \]

7. **Solve each equation.**
   
   
   
   
   \[ t = \frac{5}{8} \]

The solutions are \( \frac{5}{8} \) second and 2 seconds. The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

### Concept Check

1. **Explain how to determine which values should be chosen for \( m \) and \( n \) when factoring a polynomial of the form \( ax^2 + bx + c \).**

2. **OPEN ENDED** Write a trinomial that can be factored using a pair of numbers whose sum is 9 and whose product is 14. **Sample answer:** \( 2x^2 + 9x + 7 \)

3. **FIND THE ERROR** Dasan and Craig are factoring \( 2x^2 + 11x + 18 \).

   **Dasan**
   
   
   
   
   **Craig**
   
   
   
   

Who is correct? Explain your reasoning. **Craig; see margin for explanation.**

### Guided Practice

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.  

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – 9</td>
<td>1 – 3</td>
</tr>
<tr>
<td>10 – 12</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
</tr>
</tbody>
</table>

**Answers**

3. **When factoring a trinomial of the form** \( ax^2 + bx + c \), where \( a \neq 1 \), you must find the factors of \( ac \), not of \( c \).  

16. \( (2p + 3)(3p - 2) \)

17. \( (5d - 4)(d + 2) \)

18. \( \text{prime} \)

19. \( (3g - 2)(3g - 2) \)

20. \( (2a + 3)(a - 6) \)

21. \( (x - 4)(2x + 5) \)

22. \( (5c - 7)(c - 2) \)

23. \( \text{prime} \)

24. \( (2y - 3)(4y + 3) \)

25. \( (5n + 2)(2n - 3) \)

26. \( (5z + 9)(3z - 2) \)

27. \( (2x + 3)(7x - 4) \)

28. \( 2(3r + 2)(r - 3) \)

29. \( 5(3x + 2)(2x - 3) \)

30. \( (3x + 5y)(3x + 5y) \)

31. \( (12a - 5b)(3a + 2b) \)
Solve each equation. Check your solutions.

10. \(3x^2 + 11x + 6 = 0\)
11. \(10p^2 = 19p + 7 = 0\)
12. \(6n^2 + 7n = 20\)

13. **GYMNASTICS** When a gymnast making a vault leaves the horse, her feet are 8 feet above the ground traveling with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time \(t\) in seconds it takes for the gymnast’s feet to reach the mat. (Hint: Let \(h = 0\), the height of the mat.) **1 s**

**Practice and Apply**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

14. \(2x^2 + 7x + 5\)
15. \(3x^2 + 5x + 2\)
16. \(6y^2 + 5y - 6\)
17. \(5x^2 + 6x - 8\)
18. \(8x^2 - 19x + 9\)
19. \(9x^2 - 12x + 4\)
20. \(2x^2 - 9a - 18\)
21. \(2x^2 - 3x - 20\)
22. \(5x^2 - 17x + 14\)
23. \(3y^2 - 25y + 16\)
24. \(8y^2 - 6y - 9\)
25. \(10h^2 - 11n - 6\)
26. \(15z^2 - 17z + 18\)
27. \(14x^2 + 13x - 12\)
28. \(6x^2 - 14x - 12\)
29. \(30x^2 - 25x - 30\)

**CRITICAL THINKING** Find all values of \(x\) that each trinomial can be factored as two binomials using integers.

30. \(2x^2 + x + 12\)
31. \(2x^2 + x + 15\)
32. \(2x^2 + 12x + k, k > 0\)
33. \(2x^2 + 10\), \(14\), \(11\), \(10\), \(16\), \(18\)
34. \(2x^2 + 12x + k, k > 0\)
35. \(27x^2 + 27x + 10 = 0\)
36. \(3x^2 - 5x - 12 = 0\)
37. \(24x^2 - 11x - 3 = 3x\)
38. \(17x^2 - 11x + 2 = 2x\)
39. \(14y^2 = 25x + 25\)
40. \(12a^2 - 13a = 35\)
41. \(6x^2 - 14x = 12\)
42. \(21x^2 - 6x = 15x\)
43. \(24x^2 - 30x - 8 = -2x\)
44. \(24x^2 - 46x = 18\)

**GEOMETRY** For Exercises 49 and 50, use the following information.

A rectangle with an area of 35 square inches is formed by cutting off strips of equal width from a rectangular piece of paper. 

49. Find the width of each strip. **1 in.** 
50. Find the dimensions of the new rectangle. **5 in. by 7 in.** 

51. **CLIFF DIVING** Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below? **2.5 s**

**Enrichment, p. 546**

**Area Models for Quadratic Trinomials**

After you have factored a quadratic trinomial, you can use the factored to draw geometric models of the trinomial.

\[a = \text{length} \times \text{width} \times \text{height}\]

To draw a rectangular model, the value \(a\) not used for \(b\) or \(c\) and the length \(a\) times the width \(b\) times the width \(c\) to the height of the model.

To draw a triangular model, the area of the triangle is one-half the base times the height. The area of the circle must also be drawn to represent the base side of the rectangular model.

**Study Guide and Intervention, p. 541 (shown) and p. 542**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

1. \(2y^2 + 10y + 3\)
2. \(2y^2 + 9y - 2\)
3. \(2y^2 + 11y + 10\)
4. \(2y^2 + 5y - 3\)
5. \(y^2 + 10y + 21\)
6. \(y^2 + 3y + 2\)
7. \(y^2 - 9y - 22\)
8. \(y^2 - 10y + 21\)
9. \(2y^2 - 11y + 14\)
10. \(2y^2 - 13y - 24\)

**Skills Practice, p. 543 and Practice, p. 544 (shown)**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

1. \(3x^2 + 6x - 2\)
2. \(3y^2 + 4y - 2\)
3. \(3x^2 + 11x - 10\)
4. \(3x^2 + 13x + 4\)
5. \(3x^2 + 13x + 4\)
6. \(3x^2 + 11x - 10\)
7. \(3x^2 + 11x - 10\)
8. \(3x^2 + 13x + 4\)

**Reading to Learn Mathematics, p. 545**

Pre-Activity How can algebra tiles be used to factor \(ax^2 + bx + c\)?

Read the introduction to Lesson 9-4 on the top of page 499 in your textbook.

1. Place the two \(a\) tiles on the product mat and arrange the \(x\) tiles into a rectangular array. 
2. What is the second step? 
3. Arrange the seven \(a\) tiles to complete the rectangle.

Lesson 9-4 Factoring Trinomials: \(ax^2 + bx + c\) 499
52. CLIMBING Damaris launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge of a building she wants to scale. As it falls, the hook anchors on the ledge, which is 30 feet above the ground. How long was the hook in the air? 3 s

53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How can algebra tiles be used to factor $2x^2 + 7x + 6$?
Include the following in your answer:
- the dimensions of the rectangle formed, and
- an explanation, using words and drawings, of how this geometric guess-and-check process of factoring is similar to the algebraic process described on page 495.

54. What are the solutions of $2p^2 - p - 3 = 0$? D
   A $-\frac{2}{3}$ and 1 B $\frac{2}{3}$ and $-1$ C $-\frac{3}{2}$ and 1 D $\frac{3}{2}$ and $-1$

55. Suppose a person standing atop a building 398 feet tall throws a ball upward. If the person releases the ball 4 feet above the top of the building, the ball's height $h$, in feet, after $t$ seconds is given by the equation $h = -16t^2 + 48t + 402$. After how many seconds will the ball be 338 feet from the ground? B
   A 3.5 B 4 C 4.5 D 5

56. $a^2 - 4a - 21$  57. $t^2 + 2t + 2$  58. $a^2 + 15a + 44$
   $(a + 3)(a - 7)$  prime  $(a + 4)(a + 11)$

59. $\left\{-\frac{7}{5}, 4\right\}$
60. $\left\{\frac{3}{2}, -\frac{2}{3}\right\}$

61. $12u = u^2$ (0, 12)

62. BUSINESS Jake's Garage charges $83 for a two-hour repair job and $185 for a five-hour repair job. Write a linear equation that Jake can use to bill customers for repair jobs of any length of time. (Lesson 5-3) $y = 34x + 15$

**Maintain Your Skills**

**Mixed Review**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lesson 9-3)

63. $\frac{a^2 - 4a - 21}{(a + 3)(a - 7)}$  57. $t^2 + 2t + 2$  58. $a^2 + 15a + 44$
64. $\frac{(a + 4)(a + 11)}{}$

Solve each equation. Check your solutions. (Lesson 9-2)
65. $(y - 4)(5y + 7) = 0$  60. $(2x + 9)(3x + 2) = 0$
66. $\frac{12u}{u^2}$ (0, 12)
61. $12u = u^2$

62. BUSINESS Jake's Garage charges $83 for a two-hour repair job and $185 for a five-hour repair job. Write a linear equation that Jake can use to bill customers for repair jobs of any length of time. (Lesson 5-3) $y = 34x + 15$

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find the principal square root of each number.

To review square roots, see Lesson 2-7.

63. $\sqrt{16} = 4$  64. $\sqrt{49} = 7$  65. $\sqrt{36} = 6$  66. $\sqrt{25} = 5$
67. $\sqrt{100} = 10$  68. $\sqrt{121} = 11$  69. $\sqrt{169} = 13$  70. $\sqrt{225} = 15$

**Guess (2x + 1)(x + 3) incorrect because 8 x tiles are needed to complete the rectangle**
Factoring Differences of Squares

What You’ll Learn

- Factor binomials that are the differences of squares.
- Solve equations involving the differences of squares.

How can you determine a basketball player’s hang time?

A basketball player’s hang time is the length of time he is in the air after jumping. Given the maximum height \( h \) a player can jump, you can determine his hang time \( t \) in seconds by solving \( 4t^2 - h = 0 \). If \( h \) is a perfect square, this equation can be solved by factoring using the pattern for the difference of squares.

**Algebra Activity**

**Difference of Squares**

**Step 1** Use a straightedge to draw two squares similar to those shown below. Choose any measures for \( a \) and \( b \).

\[
\begin{array}{c}
| a \quad b \\
\hline
| a \\
\end{array}
\]

Notice that the area of the large square is \( a^2 \), and the area of the small square is \( b^2 \).

**Step 2** Cut the small square from the large square.

The area of the remaining irregular region is \( a^2 - b^2 \).

**Step 3** Cut the irregular region into two congruent pieces as shown below.

**Step 4** Rearrange the two congruent regions to form a rectangle with length \( a + b \) and width \( a - b \).

Make a Conjecture

1. Write an expression representing the area of the rectangle. \((a + b)(a - b)\)
2. Explain why \( a^2 - b^2 = (a + b)(a - b) \). Since \( a^2 - b^2 \) and \((a + b)(a - b)\) describe the same area, \( a^2 - b^2 = (a + b)(a - b) \).

Factoring Differences

A geometric model can be used to factor the difference of squares.

Study Tip

Look Back
To review the product of a sum and a difference, see Lesson 8-8.

**Lesson 9-5** Factoring Differences of Squares

**Factor** \( a^2 - b^2 \) A geometric model can be used to factor the difference of squares.

**Mathematical Background**

Notes are available for this lesson on p. 472D.

**Building on Prior Knowledge**

In Chapter 8, students learned about a special binomial product called a difference of squares. Specifically, a difference of squares is the product of two binomials of the form \( (a + b)(a - b) \), and the product is of the form \( a^2 - b^2 \). In this lesson, students will learn to factor differences of squares, and use the factoring of differences of squares to solve equations.

Ask students:

- What is a perfect square?
  A perfect square is a rational number whose square root is a rational number.
- Is \( 4t^2 \) a perfect square? If so, what is its principal square root? Yes, the principal square root is \( 2t \).
- If a basketball player can jump 4 feet, what would be her hang time? 1 second

**Resource Manager**

**Workbook and Reproducible Masters**

- Chapter 9 Resource Masters
  - Study Guide and Intervention, pp. 547–548
  - Skills Practice, p. 549
  - Practice, p. 550
  - Reading to Learn Mathematics, p. 551
  - Enrichment, p. 552
  - Assessment, p. 574
- Prerequisite Skills Workbook, pp. 13–14

- Graphing Calculator and Spreadsheet Masters, p. 40
- Parent and Student Study Guide Workbook, p. 72
- School-to-Career Masters, p. 18
- Science and Mathematics Lab Manual, pp. 71–76
- Teaching Algebra With Manipulatives Masters, pp. 24, 167

**Transparencies**

- 5-Minute Check Transparency 9-5
- Answer Key Transparencies

**Technology**

- Interactive Chalkboard
2 Teach

FACTOR $a^2 - b^2$

Teaching Tip Students may check their factoring by multiplying the factors using the FOIL method. The first-degree term will always drop out when the product is a difference of squares.

1. Factor each binomial.
   a. $m^2 - 64 \ (m + 8)(m - 8)$
   b. $16y^2 - 81z^2 \ (4y + 9z)(4y - 9z)$

2. Factor $3b^3 - 27b$.
   $3b(b + 3)(b - 3)$

Teaching Tip Students should notice that when the difference of squares factoring technique has been applied once, one of the factors should be prime.

3. Factor $4y^4 - 2500$.
   $4(y^2 + 25)(y + 5)(y - 5)$

4. Factor $6x^3 + 30x^2 - 24x - 120$.
   $6(x + 2)(x - 2)(x + 5)$

Study Tip

Common Misconception
Remember that the sum of two squares, like $x^2 + 9$, is not factorable using the difference of squares pattern. $x^2 + 9$ is a prime polynomial.

Key Concept

Difference of Squares

- Symbols $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$
- Example $x^2 - 9 = (x + 3)(x - 3)$ or $(x - 3)(x + 3)$

We can use this pattern to factor binomials that can be written in the form $a^2 - b^2$.

Example 1 Factor the Difference of Squares

Factor each binomial.

a. $n^2 - 25$
   
   $n^2 - 25 = n^2 - 5^2$
   
   $= (n + 5)(n - 5)$
   
   Factor the difference of squares.

b. $36y^2 - 49z^2$
   
   $36y^2 - 49z^2 = (6y)^2 - (7z)^2$
   
   $= (6y + 7z)(6y - 7z)$
   
   Factor the difference of squares.

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

Example 2 Factor Out a Common Factor

Factor $48a^2 - 12a$.

$48a^2 - 12a = 12a(4a^2 - 1)$

The GCF of $48a^2$ and $-12a$ is $12a$.

$= 12a[(2a)^2 - 1^2]$  \hspace{1cm} 4a^2 = 2a \cdot 2a and 1 \cdot 1

$= 12a(2a + 1)(2a - 1)$

Factor the difference of squares.

Occasionally, the difference of squares pattern needs to be applied more than once to factor a polynomial completely.

Example 3 Apply a Factoring Technique More Than Once

Factor $2x^4 - 162$.

$2x^4 - 162 = 2(x^4 - 81)$

The GCF of $2x^4$ and $-162$ is 2.

$= 2[(x^2)^2 - 9^2]$  \hspace{1cm} $x^2 = x \cdot x$ and $81 = 9 \cdot 9$

$= 2(x^2 + 9)(x^2 - 9)$

Factor the difference of squares.

$= 2(x^2 + 9)(x^2 - 3^2)$  \hspace{1cm} $x^2 - 3$ is the common factor.

$= 2(x^2 + 9)(x + 3)(x - 3)$

Factor the difference of squares, $x^2 - 1$, into $(x + 1)(x - 1)$.

Example 4 Apply Several Different Factoring Techniques

Factor $5x^3 + 15x^2 - 5x - 15$.

$5x^3 + 15x^2 - 5x - 15$

$= 5[x^3 + 3x^2 - x - 3]$  \hspace{1cm} Original polynomial

$= 5[(x^3 - x) + (3x^2 - 3)]$  \hspace{1cm} Factor the GCF.

$= 5[x(x^2 - 1) + 3(x^2 - 1)]$  \hspace{1cm} Group terms with common factors.

$= 5(x^3 - 1)(x + 3)$  \hspace{1cm} Factor each grouping.

$= 5(x - 1)(x + 1)(x + 3)$  \hspace{1cm} $x^2 - 1$ is the common factor.

$= 5(x + 1)(x - 1)(x + 3)$

Factor the difference of squares, $x^2 - 1$, into $(x + 1)(x - 1)$.

Algebra Activity

Materials:
- straightedge, scissors

- Using graph paper, students are more likely to draw straight squares, which will make the final product appear more like a rectangle.
- Make sure students label their figures as shown. Explain that the sides of the original square have length of $a$, and when the $b$ square is cut out, the remaining sides have lengths of $a - b$. 

502 Chapter 9 Factoring
**Solve Equations by Factoring**

You can apply the Zero Product Property to an equation that is written as the product of any number of factors set equal to 0.

### Example 5: Solve Equations by Factoring

Solve each equation by factoring. Check your solutions.

a. \[ p^2 - \frac{9}{16} = 0 \]
   
   \[ p^2 - \frac{9}{16} = 0 \quad \text{Original equation} \]
   
   \[ p^2 - \left(\frac{3}{4}\right)^2 = 0 \quad p^2 - p \cdot p \quad \text{Factor the difference of squares.} \]
   
   \[ \left( p + \frac{3}{4} \right) \left( p - \frac{3}{4} \right) = 0 \quad \text{Zero Product Property} \]
   
   \[ p = -\frac{3}{4} \quad \text{or} \quad p = \frac{3}{4} \quad \text{Solve each equation.} \]

The solution set is \[ \left\{ -\frac{3}{4}, \frac{3}{4} \right\} \]. Check each solution in the original equation.

b. \[ 18x^3 = 50x \]

\[ 18x^3 = 50x \quad \text{Original equation} \]

\[ 18x^3 - 50x = 0 \quad \text{Subtract 50x from each side.} \]

\[ 2x(9x^2 - 25) = 0 \quad \text{The GCF of } 18x^3 \text{ and } -50x \text{ is } 2x. \]

\[ 2x(3x + 5)(3x - 5) = 0 \quad 9x^2 - 3x \cdot 3x \quad 25 = 5 \cdot 5 \]

Applying the Zero Product Property, set each factor equal to 0 and solve the resulting three equations.

\[ 2x = 0 \quad \text{or} \quad 3x + 5 = 0 \quad \text{or} \quad 3x - 5 = 0 \]

\[ x = 0 \quad 3x = -5 \quad 3x = 5 \]

\[ x = -\frac{5}{3} \quad x = \frac{5}{3} \]

The solution set is \[ \left\{ -\frac{5}{3}, 0, \frac{5}{3} \right\} \]. Check each solution in the original equation.

### Example 6: Use Differences of Two Squares

#### Extended-Response Test Item

A corner is cut off a 2-inch by 2-inch square piece of paper. The cut is \( x \) inches from a corner as shown.

a. Write an equation in terms of \( x \) that represents the area \( A \) of the paper after the corner is removed.

b. What value of \( x \) will result in an area that is \( \frac{3}{4} \) of the area of the original square piece of paper? Show how you arrived at your answer.

---

**Teaching Tip**

Students may not be used to thinking of fractions as perfect squares. Remind them that if both the numerator and denominator are perfect squares, then the fraction itself is a perfect square. Also, if students are uncomfortable finding square roots of fractions, point out the Study Tip in the text.

**5** Solve each equation by factoring. Check your solutions.

a. \[ q^2 - \frac{4}{25} = 0 \quad \left\{ \frac{2}{5}, -\frac{2}{5} \right\} \]

b. \[ 48y^3 = 3y \quad \left\{ \frac{1}{4}, \frac{1}{4} \right\} \]

**6** **Extended Response**

A square with side length \( x \) is cut from the right triangle shown below.

a. Write an equation in terms of \( x \) that represents the area \( A \) of the triangle after the corner is removed. \( A = 64 - x^2 \)

b. What value of \( x \) will result in a triangle that is \( \frac{3}{4} \) of the area of the original triangle? Show how you arrived at your answer. \( 4 \)
3 Practice/Apply

Study Notebook

Have students—
• add the definitions/examples of the vocabulary terms to their
Vocabulary Builder worksheets for Chapter 9.
• include explanations on how to factor differences of squares.
• include any other item(s) that they find helpful in mastering the skills in this lesson.

Daily Intervention

FIND THE ERROR
Make sure students can explain what Jessica did wrong. Stress that Jessica’s error is a common one. Ask students what they can do to avoid making the same mistake themselves.

About the Exercises...

Organization by Objective
• Factor $a^2 - b^2$: 16–33
• Solve Equations by Factoring: 34–45

Odd/Even Assignments
Exercises 16–45 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 17–31 odd, 35–43 odd, 46, 47, 49, 51–70
Average: 17–43 odd, 46, 47, 49, 51–70
Advanced: 16–50 even, 51–64 (optional: 65–70)

Check for Understanding

Concept Check
1. Each term of the binomial is a perfect square, and the binomial can be written as a difference of terms.
3. Yes; $3n^2 - 48 = 3(n^2 - 16) = 3(n + 4)(n - 4)$.

Guided Practice

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. 8. $2(4x^2 + y^2)(2x + y)(2x - y)$

5. $n^2 - 81 = (n + 9)(n - 9)$
6. $4 - 9a^2 = 4 - 3a(2 + 3a)$
7. $2x^5 - 98x^3 = 2x(x + 7)(x - 7)$
8. $32x^4 - 2y^4$
9. $4t^2 - 27 = 3t(2x + 3)(x - 3)$
10. $x^3 - 3x^2 - 9x + 27 = (x + 3)(x - 3)(x - 3)$

Solve each equation by factoring. Check your solutions.
11. $4y^2 = 25$ 12. $17 - 68k^3 = 0$
13. $x^2 - \frac{1}{36} = 0$ 13. $121a = 49a^3$

FIND THE ERROR

Jessica did wrong. Stress that Jessica’s error is a common one. Have students—
• include any other item(s) that they think helpful in mastering the skills in this lesson.

Differentiated Instruction

Intrapersonal  Consider having students complete the Check for Understanding problems on one day, and then check their own work on the next day, examining their work and answers. Letting their own work sit for a day often allows students to see mistakes or problems in their work that they otherwise might not have noticed. It may also give students more of a chance to ask for help on difficult problems.

Translate the verbal statement.

$A = \frac{7}{9}A_o$
$4 - \frac{1}{2}x^2 = \frac{7}{9}$
$A = 4 - \frac{1}{2}x^2$ and $A_o$ is 4.

Simplify.

$4 - \frac{1}{2}x^2 = \frac{28}{9}$
$\frac{8}{9} - \frac{1}{2}x^2 = 0$
$16 - 9x^2 = 0$

Multiply each side by 18 to remove fractions.

$(4 + 3x)(4 - 3x) = 0$

Factor the difference of squares.

$4 + 3x = 0$ or $4 - 3x = 0$

Zero Product Property

Solve each equation.

Since length cannot be negative, the only reasonable solution is $\frac{4}{3}$.
Answer

46. Use factoring by grouping.

\[ a^2 - b^2 = a^2 + ab + 0ab - b^2 \]

\[ = (a^2 + ab) + (0ab - b^2) \]

\[ = a(a + b) - b(a + b) \]

\[ = (a - b)(a + b) \]

47. BOATING

The United States Coast Guard’s License Exam includes questions dealing with the breaking strength of a line. The basic breaking strength \( b \) in pounds for a natural fiber line is determined by the formula \( 900b^2 = k \), where \( c \) is the circumference of the line in inches. What circumference of natural line would have 3600 pounds of breaking strength? 2 in.

48. AERODYNAMICS

The formula for the pressure difference \( P \) above and below a wing is described by the formula \( P = \frac{2\pi d^2}{2\pi d^2} \), where \( d \) is the density of the air, \( \nu_1 \) is the velocity of the air passing above, and \( \nu_2 \) is the velocity of the air passing below. Write this formula in factored form.

\[ P = \frac{1}{2}d(\nu_1 - \nu_2)(\nu_1 - \nu_2) \]

49. LAW ENFORCEMENT

If a car skids on dry concrete, police can use the formula \( \frac{1}{2}d^2 = d \) to approximate the speed \( s \) of a vehicle in miles per hour given the length \( d \) of the skid marks in feet. If the length of skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied? 36 mph

50. PACKAGING

The width of a box is 3 inches more than its length. The height of the box is 1 inch less than its length. If the box has a volume of 72 cubic inches, what are the dimensions of the box? 3 in. by 12 in. by 2 in.
51. The flaw is in line 5. Since \( a = b \), \( a - b = 0 \). Therefore dividing by \( a - b \) is dividing by zero, which is undefined.

51. CRITICAL THINKING The following statements appear to prove that 2 is equal to 1. Find the flaw in this “proof.”

Suppose \( a \) and \( b \) are real numbers such that \( a = b, a \neq 0, b \neq 0 \).

(1) \( a = b \) \hspace{1cm} \text{Given.}
(2) \( a^2 = ab \) \hspace{1cm} \text{Multiply each side by } a.
(3) \( a^2 - b^2 = ab - b^2 \) \hspace{1cm} \text{Subtract } b^2 \text{ from each side.}
(4) \( (a - b)(a + b) = b(a - b) \) \hspace{1cm} \text{Factor.}
(5) \( a + b = b \) \hspace{1cm} \text{Divide each side by } a - b.
(6) \( a + a = a \) \hspace{1cm} \text{Substitution Property, } a = b
(7) \( 2a = a \) \hspace{1cm} \text{Combine like terms.}
(8) \( 2 = 1 \) \hspace{1cm} \text{Divide each side by } a.

52. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How can you determine a basketball player’s hang time?

Include the following in your answer:

• a maximum height that is a perfect square and that would be considered a reasonable distance for a student athlete to jump, and
• a description of how to find the hang time for this maximum height.

53. What is the factored form of \( 25b^2 - 1 \)?

(A) \( (5b - 1)(5b + 1) \) \hspace{1cm} (B) \( (5b - 1)(5b - 1) \) \hspace{1cm} (C) \( (5b + 1)(5b + 1) \) \hspace{1cm} (D) \( (25b + 1)(b - 1) \)

54. GRID IN In the figure, the area between the two squares is 17 square inches. The sum of the perimeters of the two squares is 68 inches. How many inches long is a side of the larger square? \( 9 \) in.

Answers

52. Answers should include the following:

• 1 foot

• To find the hang time of a student athlete who attains a maximum height of 1 foot, solve the equation \( 4t^2 - 1 = 0 \). You can factor the left side using the difference of squares pattern since \( 4t^2 \) is the square of \( 2t \) and 1 is the square of 1. Thus the equation becomes \( (2t - 1)(2t + 1) = 0 \). Using the Zero Product Property, each factor can be set equal to zero, resulting in two solutions, \( t = -\frac{1}{2} \) and \( t = \frac{1}{2} \). Since time cannot be negative, the hang time is \( \frac{1}{2} \) second.

53. What is the factored form of \( 25b^2 - 1 \)?

(A) \( (5b - 1)(5b + 1) \) \hspace{1cm} (B) \( (5b - 1)(5b - 1) \) \hspace{1cm} (C) \( (5b + 1)(5b + 1) \) \hspace{1cm} (D) \( (25b + 1)(b - 1) \)

54. GRID IN In the figure, the area between the two squares is 17 square inches. The sum of the perimeters of the two squares is 68 inches. How many inches long is a side of the larger square? \( 9 \) in.
The Language of Mathematics

Mathematics is a language all its own. As with any language you learn, you must read slowly and carefully, translating small portions of it at a time. Then you must reread the entire passage to make complete sense of what you read.

In mathematics, concepts are often written in a compact form by using symbols. Break down the symbols and try to translate each piece before putting them back together. Read the following sentence:

\[ a^2 + 2ab + b^2 = (a + b)^2 \]

*The trinomial \( a \) squared plus twice the product of \( a \) and \( b \) plus \( b \) squared equals the square of the binomial \( a + b \).*

Below is a list of the concepts involved in that single sentence.

- The letters \( a \) and \( b \) are variables and can be replaced by monomials like 2 or \( 3x \) or by polynomials like \( x + 3 \).
- The square of the binomial \( a + b \) means \( (a + b)(a + b) \). So, \( a^2 + 2ab + b^2 \) can be written as the product of two identical factors, \( a + b \) and \( a + b \).

Now put these concepts together. The algebraic statement \( a^2 + 2ab + b^2 = (a + b)^2 \) means that any trinomial that can be written in the form \( a^2 + 2ab + b^2 \) can be factored as the square of a binomial using the pattern \( (a + b)^2 \).

When reading a lesson in your book, use these steps.

- Read the “What You’ll Learn” statements to understand what concepts are being presented.
- Skim to get a general idea of the content.
- Take note of any new terms in the lesson by looking for highlighted words.
- Go back and reread in order to understand all of the ideas presented.
- Study all of the examples.
- Pay special attention to the explanations for each step in each example.
- Read any study tips presented in the margins of the lesson.

Reading to Learn

2. GCF, perfect square trinomial; \( x^2 + bx + c, ax^2 + bx + c \)

Turn to page 508 and skim Lesson 9-6.

1. List three main ideas from Lesson 9-6. Use phrases instead of whole sentences. **See margin.**

2. What factoring techniques should be tried when factoring a trinomial?

3. What should you always check for first when trying to factor any polynomial? **a greatest common factor**

4. Translate the symbolic representation of the Square Root Property presented on page 511 and explain why it can be applied to problems like \( (a + 4)^2 = 49 \) in Example 4a. **See margin.**

Answers

1. (1) explains how to factor a perfect square trinomial; (2) summarizes methods used to factor polynomials; (3) explains how to solve equations involving perfect squares using the Square Root Property

4. For any number \( n \), where \( n \) is positive, the square of \( x \) equals \( n \), then \( x \) equals plus or minus the square root of \( n \). This property can be applied to the equation \( (a + 4)^2 = 49 \) since the variable \( x = a + 4 \) and \( n = 49 \) in the equation \( x^2 = n \).
**Focus**

*What can factoring be used to design a pavilion?*

**Vocabulary**
- perfect square trinomials

**How**

The senior class has decided to build an outdoor pavilion. It will have an 8-foot by 8-foot portrayal of the school’s mascot in the center. The class is selling bricks with students’ names on them to finance the project. If they sell enough bricks to cover 80 square feet and want to arrange the bricks around the art, how wide should the border of bricks be? To solve this problem, you would need to solve the equation $(8 + 2x)^2 = 144$.

**FACTOR PERFECT SQUARE TRINOMIALS**

Numbers like 144, 16, and 49 are perfect squares, since each can be expressed as the square of an integer.

$$144 = 12 \cdot 12 \
16 = 4 \cdot 4 \
49 = 7 \cdot 7 \\
$$

Products of the form $(a + b)^2$ and $(a - b)^2$, such as $(8 + 2x)^2$, are also perfect squares. Recall that these are special products that follow specific patterns.

$$(a + b)^2 = (a + b)(a + b) \
= a^2 + ab + ab + b^2 \
= a^2 + 2ab + b^2 \\
(a - b)^2 = (a - b)(a - b) \
= a^2 - ab - ab + b^2 \
= a^2 - 2ab + b^2$$

These patterns can help you factor perfect square trinomials, trinomials that are the square of a binomial.

<table>
<thead>
<tr>
<th>Squaring a Binomial</th>
<th>Factoring a Perfect Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x + 7)^2 = x^2 + 2(x)(7) + 7^2$</td>
<td>$x^2 + 14x + 49 = x^2 + 2(x)(7) + 7^2$</td>
</tr>
<tr>
<td>$(3x - 4)^2 = (3x)^2 - 2(3x)(4) + 4^2$</td>
<td>$9x^2 - 24x + 16 = (3x)^2 - 2(3x)(4) + 4^2$</td>
</tr>
</tbody>
</table>

For a trinomial to be factorable as a perfect square, three conditions must be satisfied as illustrated in the example below.

- The first term must be a perfect square.
  - $4x^2 - (2x)^2$
- The middle term must be twice the product of the square roots of the first and last terms.
  - $2(2x)(5) = 20x$
- The last term must be a perfect square.
  - $25 = 5^2$
Factoring Perfect Square Trinomials

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

**Example 1**

1. **Factor Perfect Square Trinomials**

   Determine whether each trinomial is a perfect square trinomial. If so, factor it.

   a. $16x^2 + 32x + 64$
      
      $\begin{align*}
      &\text{Is the first term a perfect square?} \\
      &\text{Yes, } 16x^2 = (4x)^2. \\
      &\text{Is the last term a perfect square?} \\
      &\text{Yes, } 64 = 8^2. \\
      &\text{Is the middle term equal to } 2(4x)(8)? \\
      &\text{No, } 32x \neq 2(4x)(8).
      \end{align*}$

      $16x^2 + 32x + 64$ is not a perfect square trinomial.

   b. $9y^2 - 12y + 4$
      
      $\begin{align*}
      &\text{Is the first term a perfect square?} \\
      &\text{Yes, } 9y^2 = (3y)^2. \\
      &\text{Is the last term a perfect square?} \\
      &\text{Yes, } 4 = 2^2. \\
      &\text{Is the middle term equal to } 2(3y)(2)? \\
      &\text{Yes, } 12y = 2(3y)(2). \\
      \end{align*}$

      $9y^2 - 12y + 4$ is a perfect square trinomial.

      $9y^2 - 12y + 4 = (3y)^2 - 2(3y)(2) + 2^2 \\
      = (3y - 2)^2$ \\
      Write as $a^2 - 2ab + b^2$. \\
      Factor using the pattern.

In this chapter, you have learned to factor different types of polynomials. The Concept Summary lists these methods and can help you decide when to use a specific method.

### Concept Summary

<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Factoring Technique</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 or more</td>
<td>greatest common factor</td>
<td>$3x^3 + 6x^2 - 15x = 3x(x^2 + 2x - 5)$</td>
</tr>
<tr>
<td>2</td>
<td>difference of squares</td>
<td>$4x^2 - 25 = (2x + 5)(2x - 5)$</td>
</tr>
<tr>
<td>3</td>
<td>perfect square trinomial</td>
<td>$x^2 + 6x + 9 = (x + 3)^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$4x^2 - 4x + 1 = (2x - 1)^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x^2 - 9x + 20 = (x - 5)(x - 4)$</td>
</tr>
<tr>
<td>4 or more</td>
<td>factoring by grouping</td>
<td>$3xy - 6y + 5x - 10$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(3xy - 6y) + (5x - 10)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$3y(x - 2) + 5(x - 2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(x - 2)(3y + 5)$</td>
</tr>
</tbody>
</table>

### Teaching Tip

**Remind students to look closely at the operation sign in front of the second term of the trinomial. This sign signifies whether the factors are in the form $(a + b)^2$ or $(a - b)^2$.**

### Assessments

**During the last lesson of a chapter, it is often good to review some of the major concepts of the chapter to assess whether students have mastered the concepts. The concept summary table on Factoring Polynomials provides a perfect opportunity for review. Briefly review each factoring technique with students. After your review, you might consider giving students a quiz on the different techniques to assess student mastery.**
When there is a GCF other than 1, it is usually easier to factor it out first. Then, check the appropriate factoring methods in the order shown in the table. Continue factoring until you have written the polynomial as the product of a monomial and/or prime polynomial factors.

Example 2 Factor Completely

Factor each polynomial.

a. \(4x^2 - 36\)

First check for a GCF. Then, since the polynomial has two terms, check for the difference of squares.

\[
4x^2 - 36 = 4(x^2 - 9) \quad 4 \text{ is the GCF.}
\]

\[
= 4(x^2 - 3^2) \quad x^2 = x \cdot x \text{ and } 9 = 3 \cdot 3
\]

\[
= 4(x + 3)(x - 3) \quad \text{Factor the difference of squares.}
\]

b. \(25x^2 + 5x - 6\)

This polynomial has three terms that have a GCF of 1. While the first term is a perfect square, \(25x^2 = (5x)^2\), the last term is not. Therefore, this is not a perfect square trinomial.

This trinomial is of the form \(ax^2 + bx + c\). Are there two numbers \(m\) and \(n\) whose product is \(25 \cdot -6 = -150\) and whose sum is \(5\)? Yes, the product of \(15\) and \(-10\) is \(-150\) and their sum is \(5\).

\[
25x^2 + 5x - 6 = 25x^2 + mx + nx - 6 \quad \text{Write the pattern.}
\]

\[
= 25x^2 + 15x - 10x - 6 \quad m = 15 \text{ and } n = -10
\]

\[
= (25x^2 + 15x) + (-10x - 6) \quad \text{Group terms with common factors.}
\]

\[
= 5x(5x + 3) - 2(5x + 3) \quad \text{Factor out the GCF from each grouping.}
\]

\[
= (5x + 3)(5x - 2) \quad 5x + 3 \text{ is the common factor.}
\]

Example 3 Solve Equations with Repeated Factors

Solve \(x^2 - x + \frac{1}{4} = 0\).

\[
x^2 - x + \frac{1}{4} = 0 \quad \text{Original equation}
\]

\[
x^2 - 2(x)(\frac{1}{2}) + (\frac{1}{2})^2 = 0 \quad \text{Recognize } x^2 - x + \frac{1}{4} \text{ as a perfect square trinomial.}
\]

\[
(x - \frac{1}{2})^2 = 0 \quad \text{Factor the perfect square trinomial.}
\]

\[
x - \frac{1}{2} = 0 \quad \text{Set repeated factor equal to zero.}
\]

\[
x = \frac{1}{2} \quad \text{Solve for } x.
\]

Thus, the solution set is \(\{\frac{1}{2}\}\). Check this solution in the original equation.

SOLVE EQUATIONS WITH PERFECT SQUARES

When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

Logical If students do not understand how a second-degree equation can have only one solution, suggest that they graph a perfect square trinomial on a graphing calculator. The graph will immediately reveal how this is possible. The vertex of the graph of a perfect square trinomial equation lies on the x-axis, hence only one solution.
You have solved equations like \( x^2 - 36 = 0 \) by using factoring. You can also use the definition of square root to solve this equation.

\[
\begin{align*}
  x^2 - 36 &= 0 & \text{Original equation} \\
  x^2 &= 36 & \text{Add 36 to each side.} \\
  x &= \pm \sqrt{36} & \text{Take the square root of each side.}
\end{align*}
\]

Remember that there are two square roots of 36, namely 6 and \(-6\). Therefore, the solution set is \([-6, 6]\). This is sometimes expressed more compactly as \([\pm 6]\). This and other examples suggest the following property.

### Key Concept

#### Square Root Property

- **Symbols** For any number \( n > 0 \), if \( x^2 = n \), then \( x = \pm \sqrt{n} \).
- **Example**

  \[
  x^2 = 9 \\
  x = \pm \sqrt{9} \text{ or } 3
  \]

### Example 4

**Use the Square Root Property to Solve Equations**

Solve each equation. Check your solutions.

**a.** \((a + 4)^2 = 49\)

\[
\begin{align*}
  (a + 4)^2 &= 49 & \text{Original equation} \\
  a + 4 &= \pm \sqrt{49} & \text{Square Root Property} \\
  a + 4 &= \pm 7 \\
  a &= -4 \pm 7 & \text{Subtract 4 from each side.} \\
  a &= -4 + 7 \text{ or } a = -4 - 7 & \text{Separate into two equations.} \\
  a &= 3 \text{ or } a = -11 & \text{Simplify.}
\end{align*}
\]

The solution set is \([-11, 3]\). Check each solution in the original equation.

**b.** \(y^2 - 4y + 4 = 25\)

\[
\begin{align*}
  y^2 - 4y + 4 &= 25 & \text{Original equation} \\
  (y - 2)^2 &= 25 & \text{Recognize perfect square trinomial.} \\
  y - 2 &= \pm \sqrt{25} & \text{Square Root Property} \\
  y &= 2 \pm 5 \\
  y &= 2 \pm 5 & \text{Add 2 to each side.} \\
  y &= 2 + 5 \text{ or } y = 2 - 5 & \text{Separate into two equations.} \\
  y &= 7 \text{ or } y = -3 & \text{Simplify.}
\end{align*}
\]

The solution set is \([-3, 7]\). Check each solution in the original equation.

**c.** \((x - 3)^2 = 5\)

\[
\begin{align*}
  (x - 3)^2 &= 5 & \text{Original equation} \\
  x - 3 &= \pm \sqrt{5} & \text{Square Root Property} \\
  x &= 3 \pm \sqrt{5} & \text{Add 3 to each side.}
\end{align*}
\]

Since 5 is not a perfect square, the solution set is \([3 \pm \sqrt{5}]\). Using a calculator, the approximate solutions are 3 + \(\sqrt{5}\) or about 5.24 and 3 - \(\sqrt{5}\) or about 0.76.
3 Practice/Apply

Have students—
- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 9.
- include explanations on how to factor perfect square trinomials.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...
Organization by Objective
- Factor Perfect Square Trinomials: 17–22
- Solve Equations With Perfect Squares: 43–48

Assignment Guide
Average: 17–53 odd, 55–59, 60–80
Advanced: 18–54 even, 57, 58, 60–80

Teaching Tip
Remind students that any of the factoring methods they have studied thus far can be used in the exercises.

Check for Understanding

Concept Check
1. See margin.
2. Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
   \[ (a - b)^2 = a^2 - 2ab + b^2 \]
   \[ a^2 - 2ab - b^2 = (a - b)^2, b \neq 0 \]
3. OPEN ENDED Write a polynomial that requires at least two different factoring techniques to factor it completely. Sample answer: \( x^2 + 5x^2 - 4x - 20 \)

Guided Practice

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

4. \( y^2 + 8y + 16 \) yes; \( (y + 4)^2 \)
5. \( 9x^2 - 30x + 10 \) no

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.
6. \( 2x^2 + 18 \) prime
7. \( c^2 - 5c + 6 \) \( (c - 3)(c - 2) \)
8. \( 5a^3 - 80a \) \( 5a(a + 4)(a - 4) \)
9. \( 8x^2 - 18x - 35 \) \( (2x - 7)(4x + 5) \)
10. \( 9g^2 + 12g - 4 \) prime
11. \( 3n^3 + 2m^2n - 12m - 8n \) \( (m - 2)(m + 2)(3m + 2n) \)

Solve each equation. Check your solutions.
12. \( 4y^2 + 24y + 36 = 0 \) \( -3 \)
13. \( 3n^2 = 48 \) \( \pm 4 \)
14. \( a^2 - 6a + 9 = 16 \) \( -1, 7 \)
15. \( (m - 5)^2 = 13 \) \( 5 \pm \sqrt{13} \)

Application

16. HISTORY Galileo demonstrated that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa. A model for the height \( h \) in feet of an object dropped from an initial height \( h_0 \) in feet is \( h = 16t^2 + h_0 \), where \( t \) is the time in seconds after the object is dropped. Use this model to determine approximately how long it took for the objects to hit the ground if Galileo dropped them from a height of 180 feet. about \( 3.35 \) s

Answers

1. Determine if the first term is a perfect square. Then determine if the last term is a perfect square. Finally, check to see if the middle term is equal to twice the product of the square roots of the first and last terms.
   17. no
   18. yes; \( (a - 12)^2 \)
   19. yes; \( (2y - 11)^2 \)
   20. no
   21. yes; \( (3n + 7)^2 \)
   22. yes; \( (5a - 12b)^2 \)
Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

25. \(4k^2 - 100 \) 26. \(4k + 5)(k - 5)\)
27. \(x^2 + 6x - 9\) 28. \(50x^2 + 40g + 8 \) \(2(5g + 2)^2\)
29. \(9f^2 + 66f - 48\) 30. \(3f(3f - 2)(f + 8)\)
31. \(20r^2 + 34n + 6 \) \(2(5n + 1)(2n + 3)\)
32. \(5y^2 - 90 \) \(5(y^2 - 18)\)
33. \(24x^3 - 78x^2 + 45x\) \(3(4x^3 - 3x - 2)(x - 5)\)
34. \(18y^5 - 48y + 32 \) \(2(3y - 4)^2\)
35. \(90x^8 - 27x^2 - 75 - 3x(3g - 5)^2\)
36. \(45c^2 - 32cd \) \(c(45c - 32d)\)
37. \((a^2 + 2)(4a + 3b^2)\)

**41. GEOMETRY** The volume of a rectangular prism is \(x^2y^3 - 63xy^2 + 7x^2y\) cubic meters. Find the dimensions of the prism if it can be represented by binomials with integral coefficients. \(x - 3y\), \(x + 3y\), \(xy + 7\)

42. \(8x^2 - 22x + 14\) \(2x^2 - 4x + 3\) \(2x^2 - 5x + 3\) \(2x^2 - 7x + 7\)

**42. GEOMETRY** If the area of the square shown below is \(16x^2 - 56x + 49\) square inches, what is the area of the rectangle in terms of \(x\)?

Solve each equation. Check your solutions.

43. \(3x^2 + 24x + 48 = 0\) \(\{4\} -4\)
44. \(7x^2 = 70x - 175\) \(\{5\} 5\)
45. \(49x^2 + 16 = 56a\) \(\{3\} 3\)
46. \(18g^2 + 24y + 8 = 0\) \(\{-\frac{2}{3}\} -\frac{2}{3}\)
47. \(y^2 - \frac{2}{3}x^2 = 1\) \(\{\frac{1}{9}\} \frac{1}{9}\)
48. \(a^2 + 4b^2 + \frac{4}{3} = 0\) \(\{0\} 0\)
49. \(z^2 + 2x + 1 = 16\) \(\{5, 3\} 5, 3\)
50. \(x^2 + 10x + 25 = 81\) \(\{-14, 4\} -14, 4\)
51. \((y - 8)^2 = 7\) \(\{\sqrt{7}, -\sqrt{7}\} \sqrt{7}, -\sqrt{7}\)
52. \((w + 3)^2 = 2\) \(\{-3 \pm \sqrt{2}\} -3 \pm \sqrt{2}\)
53. \(p^2 + 2p + 1 = 6\) \(\{-1 \pm \sqrt{6}\} -1 \pm \sqrt{6}\)
54. \(x^2 - 12x + 36 = 11\) \(\{6 \pm \sqrt{11}\} 6 \pm \sqrt{11}\)

**FORESTRY** For Exercises 55 and 56, use the following information.

Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the **Dyke Log Rule**, \(B = \frac{L}{16}(D^2 - 8D + 16)\), where \(B\) is the number of board feet, \(D\) is the diameter in inches, and \(L\) is the length of the log in feet.

55. Write this formula in factored form. \(B = \frac{L}{16}(D - 4)^2\)
56. For logs that are 16 feet long, what diameter will yield approximately 256 board feet? 20 in.

**FREE-FALL RIDE** For Exercises 57 and 58, use the following information.

The height \(h\) in feet of a car above the exit ramp of an amusement park’s free-fall ride can be modeled by \(h = -16t^2 + s\), where \(t\) is the time in seconds after the car drops and \(s\) is the starting height of the car in feet.

57. How high above the car’s exit ramp should the ride’s designer start the drop in order for riders to experience free fall for at least 3 seconds? 144 ft
58. Approximately how long will riders be in free fall if their starting height is 160 feet above the exit ramp? 3.16 s

**www.algebra1.com/self_check_quiz**

**Lesson 9-6 Perfect Squares and Factoring 513**

**Fluent in Mathematics, p. 555**

**Ell**

**More About...**

**Free-Fall Ride**

Some amusement park free-fall rides can seat 4 passengers across per coach and reach speeds of up to 62 miles per hour.

**Source:** worldparksinfo.com

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**Study Guide and Intervention, p. 553 (shown) and p. 554**

**Factor Perfect Square Trinomials**

\[ x^2 + 10x + 25 = (x + 5)^2 \]

**Factored Form**

\[ x^2 + 10x + 25 = (x + 5)^2 \]

**Perfect Square Trinomial**

\[ x^2 + 10x + 25 = (x + 5)^2 \]

**Perfect Square Trinomial**

\[ x^2 + 10x + 25 = (x + 5)^2 \]

**Skills Practice, p. 555 and p. 556 (shown)**

**Determine whether each binomial is a perfect square trinomial. If so, factor it.**

1. \(16x^2 - 8x + 1\) \(4x(x - 1)^2\)
2. \(25a^2 + 10a + 1\) \((5a + 1)^2\)
3. \(4b^2 + 20b + 25\) \((2b + 5)^2\)
4. \(4c^2 - 12c + 9\) \((2c - 3)^2\)

**Determine whether each binomial is a perfect square trinomial. If so, factor it.**

1. \(16x^2 - 8x + 1\) \(4x(x - 1)^2\)
2. \(25a^2 + 10a + 1\) \((5a + 1)^2\)
3. \(4b^2 + 20b + 25\) \((2b + 5)^2\)
4. \(4c^2 - 12c + 9\) \((2c - 3)^2\)

**Area**

\[ 3.14 \times 10^7 \text{ m}^2 \]

**Pre-Activity**

**How can factoring be used to design a position?**

**ELL**

**Reading the Lesson**

1. **Three conditions must be met if a trinomial can be factored as a perfect square trinomial.**

2. **The height of the parabola**

3. **The maximum of the parabola**

**Getting Started**

1. **The vertex of the parabola**

2. **The zeros of the parabola**

3. **The maximum of the parabola**

**Helping You Remember**

1. **The zeros of the parabola**

2. **The y-intercept**

3. **The vertex of the parabola**

**Reading**

1. **Three conditions must be met if a trinomial can be factored as a perfect square trinomial.**

2. **The height of the parabola**

3. **The maximum of the parabola**

4. **The zeros of the parabola**

5. **The vertex of the parabola**

6. **The y-intercept**

**Lesson 9-6 Perfect Squares and Factoring**

**Enrichment, p. 558**

**Squaring Numbers: A Shortcut**

A shortcut helps you to square a perfect square-trinomial number ending in 5. The method is developed using the idea that a two-digit number may be factored as follows: 

\[ (ab + 5)^2 = (10a + 1)^2 \]

\[ = 100a^2 + 20a + 1 \]

In words: this formula states that the square of a two-digit number has two parts, the “units place” and the “hundreds place.” Then it is the two digit and 1 is the “tens digit.”

\[ \text{Examples} \]

Using the formula for \((15 + 5)^2\), find 205.

\[ 205^2 = (200 + 5)^2 \]

\[ = 40000 + 2000 + 25 \]

\[ = 42025 \]

**Helping You Remember**

1. **Determining the value of a trinomial**

2. **Checking the result of a calculation**

3. **Verifying the correctness of a solution**

**www.algebra1.com/self_check_quiz**

**Lesson 9-6 Perfect Squares and Factoring 513**
59. **HUMAN CANNONBALL** A circus acrobat is shot out of a cannon with an initial upward velocity of 64 feet per second. If the acrobat leaves the cannon 6 feet above the ground, will he reach a height of 70 feet? If so, how long will it take him to reach that height? Use the model for vertical motion. **yes; 2 s**

**CRITICAL THINKING** Determine all values of \(k\) that make each of the following a perfect square trinomial. **62. 70, –70**

60. \(x^2 + kx + 64\) **16, –16**

61. \(4x^2 + kx + 1\) **4, –4**

62. \(25x^2 + kx + 49\)

63. \(x^2 + 8x + k\) **16**

64. \(x^2 – 18x + k\) **81**

65. \(x^2 + 20x + k\) **100**

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can factoring be used to design a pavilion?

Include the following in your answer:

- an explanation of how the equation \((8 + 2x)^2 = 144\) models the given situation, and
- an explanation of how to solve this equation, listing any properties used, and an interpretation of its solutions.

67. During an experiment, a ball is dropped off a bridge from a height of 205 feet. The formula 205 = 16t^2 can be used to approximate the amount of time, in seconds, it takes for the ball to reach the surface of the river below the bridge. Find the time it takes the ball to reach the water to the nearest tenth of a second. **C 2.3 s**

68. If \(\sqrt{a^2 – 2ab + b^2} = a – b\), then which of the following statements best describes the relationship between \(a\) and \(b\)? **D**

\(A\) \(a < b\)  \(B\) \(a = b\)  \(C\) \(a > b\)  \(D\) \(a \geq b\)

**Open-Ended Assessment**

**Writing** Ask students to look at the concept summary table on page 509, and decide which factoring technique they like best. Then have students write a description of how to use their favorite technique, and why they think it is a better method as compared to one or two of the other methods.

**Assessment Options**

**Quiz (Lesson 9-6)** is available on p. 574 of the **Chapter 9 Resource Masters**.

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**Maintain Your Skills**

**Mixed Review** Solve each equation. Check your solutions. **(Lessons 9-4 and 9-5)**

69. \(s^2 = 25\) \(± 5\)

70. \(9x^2 – 16 = 0\) \(± \frac{4}{3}\)

71. \(49w^2 = 81\) \(± \frac{9}{7}\)

72. \(8k^2 + 22k – 6 = 0\)

73. \(12w^2 + 23w = –5\)

74. \(6z^2 + 7 = 17z\)

75. \((1, 4), y = 2x – 1\) \(y = \frac{1}{2}x + \frac{9}{2}\)

76. \((-4, 7), y = -\frac{2}{3}x + 7\) \(y = \frac{3}{2}x + 13\)

77. **NATIONAL LANDMARKS** At the Royal Gorge in Colorado, an inclined railway takes visitors down to the Arkansas River. Suppose the slope is 50% or \(\frac{1}{2}\) and the vertical drop is 1015 feet. What is the horizontal change of the railway? **(Lesson 5-1) 2030 ft**

Find the next three terms of each arithmetic sequence. **(Lesson 4-7)**

78. 17, 13, 9, … **9, –5, –4.5, –4, –3.5, …**

79. –5, –4.5, –4, –3.5, … **80. 45, 54, 63, 72, …**

81. 90, 99

\(\bullet (8 + 2x)^2 = 144\)

\(8 + 2x = ±12\) \(\text{Square Root Property}\)

\(8 + 2x = 12\) or \(8 + 2x = –12\) \(\text{Separate into two equations.}\)

\(2x = 4\) \(2x = –20\) \(\text{Solve each equation.}\)

\(x = 2\) \(x = –10\)

Since length cannot be negative, the border should be 2 feet wide.

---

**514 Chapter 9 Factoring**
State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. The number 27 is an example of a composite number. false, composite
2. 2x is the greatest common factor (GCF) of 12x^2 and 14xy. true
3. 66 is an example of a perfect square. false, sample answer: 64
4. 61 is a factor of 183. true
5. The prime factorization for 48 is false, 2^4 \cdot 3
6. x^2 – 25 is an example of a perfect square trinomial. false, difference of squares
7. The number 35 is an example of a prime number. true
8. Expressions with four or more unlike terms can be factored by grouping. true
9. (b – 7)(b + 7) is the factorization of a difference of squares. true

Lesson-by-Lesson Review

9-1 Factors and Greatest Common Factors

Concept Summary

- Prime number: whole number greater than 1 with exactly two factors
- Composite number: whole number greater than 1 with more than two factors
- The greatest common factor (GCF) of two or more monomials is the product of their common factors.

Example

Find the GCF of 15x^2y and 45xy^2.

15x^2y = \overset{3}{\boxed{3}} \cdot \overset{5}{\boxed{5}} \cdot x \cdot y \quad \text{Factor each number.}

45xy^2 = \overset{3}{\boxed{3}} \cdot \overset{5}{\boxed{5}} \cdot x \cdot y \quad \text{Circle the common prime factors.}

The GCF is 3 \cdot 5 \cdot x \cdot y or 15xy.

Exercises Find the prime factorization of each integer.

See Examples 2 and 3 on page 475.

11. 28 2 \cdot 7
12. 33 3 \cdot 11
13. 150 2 \cdot 3 \cdot 5^2
14. 301 7 \cdot 43
15. –83 –1 \cdot 83
16. –378 –1 \cdot 2 \cdot 3^3 \cdot 7

Find the GCF of each set of monomials. See Example 5 on page 476.

17. 35, 30 5
18. 12, 18, 40 2
19. 12ab, 4a^2b^2 4ab
20. 16mrt, 30m^2r \quad 2mr
21. 20r^2, 25mr^5 5n
22. 60r^2y^2, 35xy^3 5r

Vocabulary and Concept Check

- Composite number (p. 474)
- Factoring (p. 481)
- Prime number (p. 474)
- Factoring by grouping (p. 482)
- Greatest common factor (GCF) (p. 476)
- Prime polynomial (p. 497)
- Perfect square trinomials (p. 508)
- Prime factorization (p. 475)
- Zero Product Property (p. 483)
- Greatest common factor (GCF) (p. 476)
- Square Root Property (p. 511)
- Zero Product Property (p. 483)

Chapter 9 Resource Masters available on p. 572 of the Chapter 9 Resource Masters.
9-2

Factoring Using the Distributive Property

Concept Summary

• Find the greatest common factor and then use the Distributive Property.

• With four or more terms, try factoring by grouping.

Factoring by Grouping: \( ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y) \)

• Factoring can be used to solve some equations.

Zero Product Property: For any real numbers \( a \) and \( b \), if \( ab = 0 \), then either \( a = 0 \), \( b = 0 \), or both \( a \) and \( b \) equal zero.

Example

Factor \( 2x^2 - 3xz - 2xy + 3yz \).

\[
2x^2 - 3xz - 2xy + 3yz = (2x^2 - 3xz) + (-2xy + 3yz) = x(2x - 3z) - y(2x - 3z) = (x - y)(2x - 3z).
\]

Exercises

25. \( 2a(13b + 9c + 16a) \)

23. \( 13x + 26y \)

24. \( 6ab(4ab - 3) \)

28. \( abm - 9mn + 40ln - 15bn \)

26. \( 26ab + 18ac + 32a^2 \)

27. \( 4rs + 12ps + 2mn + 6mp \)

29. \( x(2x - 5) = 0 \)

30. \( (3u + 8)(2u - 6) = 0 \)

31. \( 4x^2 = -7x \)

32. \( x^2 + bx + c \)

33. \( x(12x + 3) \)

34. \( b^2 + 5b - 6 = (b + 6)(b - 1) \)

35. \( 18 - 9r + r^2 \)

36. \( a^2 + 6ax - 40x^2 \)

37. \( m^2 - 4mn - 32n^2 \)

38. \( y^2 + 13y + 40 = 0 \)

39. \( x^2 - 5x - 66 = 0 \)

40. \( m^2 - m - 12 = 0 \)

9-3

Factoring Trinomials: \( x^2 + bx + c \)

Concept Summary

• Factoring \( x^2 + bx + c \): Find \( m \) and \( n \) whose sum is \( b \) and whose product is \( c \).

Then write \( x^2 + bx + c \) as \((x + m)(x + n)\).

Example

Solve \( a^2 - 3a - 4 = 0 \). Then check the solutions.

\[
a^2 - 3a - 4 = 0 \quad \text{Original equation}
\]

\[
(a + 1)(a - 4) = 0 \quad \text{Factor}
\]

\[
a + 1 = 0 \quad \text{or} \quad a - 4 = 0 \quad \text{Zero Product Property}
\]

\[
a = -1 \quad a = 4 \quad \text{Solve each equation}
\]

The solution set is \(-1, 4\).

Exercises

32. \( (y + 3)(y + 4) \)

33. \( (x - 12)(x + 3) \)

34. \( 5b^2 + 5b + 6 = (b + 6)(b - 1) \)

36. \( a^2 + 6ax - 40x^2 \)

37. \( m^2 - 4mn - 32n^2 \)

38. \( y^2 + 13y + 40 = 0 \)

39. \( x^2 - 5x - 66 = 0 \)

40. \( m^2 - m - 12 = 0 \)

\{−3, 4\}

\{−5, −8\}

\{−6, 11\}
Factoring Trinomials: \( ax^2 + bx + c \)

**Concept Summary**
- Factoring \( ax^2 + bx + c \): Find \( m \) and \( n \) whose product is \( ac \) and whose sum is \( b \). Then, write as \( ax^2 + mx + nx + c \) and use factoring by grouping.

**Example**

Factor \( 12x^2 + 22x - 14 \).

First, factor out the GCF, 2: \( 12x^2 + 22x - 14 = 2(6x^2 + 11x - 7) \). In the new trinomial, \( a = 6, b = 11 \) and \( c = -7 \). Since \( b \) is positive, \( m + n \) is positive. Since \( c \) is negative, \( mn \) is negative. So either \( m \) or \( n \) is negative, but not both. Therefore, make a list of the factors of 6(−7) or −42, where one factor in each pair is negative. Look for a pair of factors whose sum is 11.

<table>
<thead>
<tr>
<th>Factors of −42</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1, 42</td>
<td>41</td>
</tr>
<tr>
<td>1, −42</td>
<td>−41</td>
</tr>
<tr>
<td>−2, 21</td>
<td>19</td>
</tr>
<tr>
<td>2, −21</td>
<td>−19</td>
</tr>
<tr>
<td>−3, 14</td>
<td>11</td>
</tr>
</tbody>
</table>

The correct factors are −3 and 14.

\[ 6x^2 + 11x - 7 = 6x^2 + mx + nx - 7 \]

Write the pattern.

\[ = 6x^2 - 3x + 14x - 7 \quad m = -3 \text{ and } n = 14 \]

Group terms with common factors.

\[ = (6x^2 - 3x) + (14x - 7) \]

Factor the GCF from each grouping.

\[ = 3x(2x - 1) + 7(2x - 1) \]

\[ = (2x - 1)(3x + 7) \]

Thus, the complete factorization of \( 12x^2 + 22x - 14 \) is \( 2(2x - 1)(3x + 7) \).

**Exercises**
- Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.
- **See Examples 1–3 on pages 496 and 497.**

42. \((2m - 3)(m + 8)\)
43. \((5r + 2)(5r + 2)\)

**Solve each equation.**
- **See Example 4 on page 497.**

47. \(2r^2 - 3r - 20 = 0\)
48. \(3a^2 - 13a + 14 = 0\)
49. \(40x^2 + 2x = 24\)

\[ \{4, \frac{3}{4} \}, \{-\frac{5}{2} \} \]

\[ \{2, \frac{7}{3} \} \]

Factoring Differences of Squares

**Concept Summary**
- Difference of Squares: \( a^2 - b^2 = (a + b)(a - b) \) or \( (a - b)(a + b) \)
- Sometimes it may be necessary to use more than one factoring technique or to apply a factoring technique more than once.

**Example**

Factor \( 3x^3 - 75x \).

\[ 3x^3 - 75x = 3x(x^2 - 25) \]

The GCF of \( 3x^3 \) and \( 75x \) is \( 3x \).

\[ = 3(x + 5)(x - 5) \]

Factor the difference of squares.

Chapter 9 Study Guide and Review
518 Chapter 9

Factoring

**Exercises** Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. *See Examples 1–4 on page 502.*

50. \(2y^3 - 128y\)
51. \(9b^2 - 20\) prime
52. \(\frac{1}{4}n^2 - \frac{9}{16}r^2\)

Solve each equation by factoring. Check your solutions. *See Example 5 on page 503.*

53. \(b^2 - 16 = 0\) \((-4, 4)\)
54. \(25 - 9y^2 = 0\) \((-\frac{5}{3}, \frac{5}{3})\)
55. \(16a^2 - 81 = 0\) \((-\frac{9}{4}, \frac{9}{4})\)
56. \(\left(\frac{1}{2}n - \frac{3}{4}\right)^2 \left(\frac{1}{2}n + \frac{3}{4}\right)^2\)

**9-6 Perfect Squares and Factoring**

**Concept Summary**

- If a trinomial can be written in the form \(a^2 + 2ab + b^2\) or \(a^2 - 2ab + b^2\), then it can be factored as \((a + b)^2\) or \((a - b)^2\), respectively.
- For a trinomial to be factorable as a perfect square, the first term must be a perfect square, the middle term must be twice the product of the square roots of the first and last terms, and the last term must be a perfect square.
- Square Root Property: For any number \(n > 0\), if \(x^2 = n\), then \(x = \pm \sqrt{n}\).

**Examples**

1. Determine whether \(9x^2 + 24xy + 16y^2\) is a perfect square trinomial. If so, factor it.
   - Is the first term a perfect square? Yes, \(9x^2 = (3x)^2\).
   - Is the last term a perfect square? Yes, \(16y^2 = (4y)^2\).
   - Is the middle term equal to \(2(3x)(4y)\)? Yes, \(24xy = 2(3x)(4y)\).
   \[9x^2 + 24xy + 16y^2 = (3x)^2 + 2(3x)(4y) + (4y)^2\]
   Write as \(a^2 + 2ab + b^2\).
   \[
   = (3x + 4y)^2
   \]
   Factor using the pattern.

2. Solve \((x - 4)^2 = 121\).
   \[
   (x - 4)^2 = 121 \quad \text{Original equation}
   
   x - 4 = \pm \sqrt{121} \quad \text{Square Root Property}
   
   x - 4 = \pm 11
   
   121 = 11 \times 11
   
   x = 4 \pm 11 \quad \text{Add 4 to each side.}
   
   x = 4 + 11 \quad \text{or} \quad x = 4 - 11 \quad \text{Separate into two equations.}
   
   = 15 \quad = -7 \quad \text{The solution set is \{-7, 15\}.}
   
**Exercises** Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. *See Example 2 on page 510.*

56. \(a^2 + 18a + 81\) \((a + 9)^2\)
57. \(9a^2 - 12k + 4\) \((3k - 2)^2\)
58. \(4 - 28r + 49r^2\) \((2 - 7r)^2\)
59. \(32m^2 - 80m + 50\) \(2(4m - 5)^2\)

Solve each equation. Check your solutions. *See Examples 3 and 4 on pages 510 and 511.*

60. \(6b^2 - 24b^3 + 24b = 0\) \((0, 2)\)
61. \(49m^2 - 126m + 81 = 0\) \(\left\{\frac{9}{7}\right\}\)
62. \((c - 9)^2 = 144\) \((-3, 21)\)
63. \(144b^2 = 36\) \(\left\{\frac{1}{2}, -\frac{1}{2}\right\}\)
Vocabulary and Concepts

1. Give an example of a prime number and explain why it is prime. Sample answer: 7; Its only factors are 1 and itself.
2. Write a polynomial that is the difference of two squares. Then factor your polynomial. Sample answer: \( n^2 - 100; (n + 10)(n - 10) \)
3. Describe the first step in factoring any polynomial. Check for a GCF other than 1 and factor it out.

Skills and Applications

Find the prime factorization of each integer.

4. 63 \( 3^2 \cdot 7 \)
5. 81 \( 3^4 \)
6. \(-210 \) \(-1 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \)

Find the GCF of the given monomials.

7. 48, 64 \( 16 \)
8. 28, 75 \( 1 \), relatively prime
9. \( 18a^2b^2, 28a^2b^2 \) \( 2a^2b^2 \)

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.

10. \( 25y^2 - 49w^2 \)
11. \( t^2 - 16t + 64 \) \( (t - 8)^2 \)
12. \( x^2 + 14x + 24 \) \( (x + 12)(x + 2) \)
13. \( 28m^2 + 18m \) \( 2m(14m + 9) \)
14. \( a^2 - 11ab + 18b^2 \)
15. \( 12x^2 + 23x - 24 \) \( (3x + 8)(4x - 3) \)
16. \( 2h^2 - 3h - 18 \) \( \text{prime} \)
17. \( 6x^3 + 15x^2 - 9x \)
18. \( 64p^2 - 63p + 16 \) \( \text{prime} \)
19. \( 2d^2 + d - 1 \) \( (2d - 1)(d + 1) \)
20. \( 36a^2b^2 - 45ab^4 \) \( 9ab^2(4a - 5b) \)
21. \( 36m^2 + 60mn + 25n^2 \) \( (6m + 5n)^2 \)
22. \( a^2 - 4 \) \( 22 - 27 \), See margin.
23. \( 4my - 20m + 3py - 15p \)
24. \( 15a^2b + 5a^2 - 10a \)
25. \( 6y^2 - 5y - 6 \)
26. \( 4x^2 - 100 \)
27. \( x^3 - 4x^2 - 9x + 36 \)

Write an expression in factored form for the area of each shaded region.

28. \( 6(x + y + 6) \)
29. \( 4r^2(4 - \pi) \)

Solve each equation. Check your solutions.

30. \( 4(x - 3)(3x + 2) = 0 \) \( \{ \frac{3}{4}, -\frac{2}{3} \} \)
31. \( 18s^2 + 72s = 0 \) \( \{ 0, -4 \} \)
32. \( 4x^2 = 36 \) \( \{ -3, 3 \} \)
33. \( t^2 + 25 = 10t \) \( \{ 5 \} \)
34. \( a^2 - 9a - 52 = 0 \) \( \{ -4, 13 \} \)
35. \( x^3 - 5x^2 - 66x = 0 \) \( \{ -6, 0, 11 \} \)
36. \( 2x^2 = 9x + 5 \) \( \{ \frac{1}{2}, 5 \} \)
37. \( 3b^2 + 6 = 11b \) \( \{ \frac{2}{3}, 3 \} \)
38. GEOMETRY A rectangle is 4 inches wide by 7 inches long. When the length and width are increased by the same amount, the area is increased by 26 square inches. What are the dimensions of the new rectangle? 6 in. by 9 in.
39. CONSTRUCTION A rectangular lawn is 24 feet wide by 32 feet long. A sidewalk will be built along the inside edges of all four sides. The remaining lawn will have an area of 425 square feet. How wide will the walk be? 3.5 ft
40. STANDARDIZED TEST PRACTICE The area of the shaded part of the square shown at the right is 98 square meters. Find the dimensions of the square. 14 m by 14 m

www.algebra1.com/chapter_test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 9 can be found on p. 572 of the Chapter 9 Resource Masters.

Chapter Tests There are six Chapter 9 Tests and an Open-Ended Assessment task available in the Chapter 9 Resource Masters.

<table>
<thead>
<tr>
<th>Chapter 9 Tests</th>
<th>Form</th>
<th>Type</th>
<th>Level</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MC</td>
<td>basic</td>
<td></td>
<td>559–560</td>
</tr>
<tr>
<td>2A</td>
<td>MC</td>
<td>average</td>
<td></td>
<td>561–562</td>
</tr>
<tr>
<td>2B</td>
<td>MC</td>
<td>average</td>
<td></td>
<td>563–564</td>
</tr>
<tr>
<td>2C</td>
<td>FR</td>
<td>average</td>
<td></td>
<td>565–566</td>
</tr>
<tr>
<td>2D</td>
<td>FR</td>
<td>average</td>
<td></td>
<td>567–568</td>
</tr>
<tr>
<td>3</td>
<td>FR</td>
<td>advanced</td>
<td></td>
<td>569–570</td>
</tr>
</tbody>
</table>

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 9 can be found on p. 571 of the Chapter 9 Resource Masters. A sample scoring rubric for these tasks appears on p. A25.

TestCheck and Worksheet Builder
This networkable software has three modules for assessment.

• Worksheet Builder to make worksheets and tests.
• Student Module to take tests on-screen.
• Management System to keep student records.

Portfolio Suggestion

Introduction Have you ever noticed that when you are learning the concepts in a chapter, such as how to factor polynomials in this chapter, that there is often more than one way to solve a problem?

Ask Students Pick a trinomial that can be factored by more than one method that you learned in this chapter, and explain how to factor it using these methods. Make sure you include a worked-out example with your descriptions in your portfolio.
These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 9 Resource Masters.

**Part 1 Multiple Choice**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which equation best describes the function graphed below? (Lesson 5-5)  
   A. \( y = \frac{-3}{5}x - 3 \)  
   B. \( y = \frac{3}{5}x - 3 \)  
   C. \( y = \frac{-5}{3}x - 3 \)  
   D. \( y = \frac{5}{3}x - 3 \)

2. The school band sold tickets to their spring concert every day at lunch for one week. Before they sold any tickets, they had $80 in their account. At the end of each day, they recorded the total number of tickets sold and the total amount of money in the band’s account.

<table>
<thead>
<tr>
<th>Day</th>
<th>Total Number of Tickets Sold</th>
<th>Total Amount in Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>12</td>
<td>$176</td>
</tr>
<tr>
<td>Tuesday</td>
<td>18</td>
<td>$224</td>
</tr>
<tr>
<td>Wednesday</td>
<td>24</td>
<td>$272</td>
</tr>
<tr>
<td>Thursday</td>
<td>30</td>
<td>$320</td>
</tr>
<tr>
<td>Friday</td>
<td>36</td>
<td>$368</td>
</tr>
</tbody>
</table>

Which equation describes the relationship between the total number of tickets sold \( t \) and the amount of money in the band’s account \( a \)? (Lesson 5-4)  
   D. \( a = \frac{1}{8}t + 80 \)  
   A. \( a = \frac{1}{8}t + 80 \)  
   C. \( a = 6t + 8 \)  
   B. \( a = 8t + 80 \)

3. Which inequality represents the shaded portion of the graph? (Lesson 6-6)  
   A. \( y \geq \frac{1}{3}x - 1 \)  
   B. \( y \leq \frac{1}{3}x - 1 \)  
   C. \( y \leq 3x + 1 \)  
   D. \( y \geq 3x - 1 \)

4. Today, the refreshment stand at the high school football game sold twice as many bags of popcorn as were sold last Friday. The total sold both days was 258 bags. Which system of equations will determine the number of bags sold today \( n \) and the number of bags sold last Friday \( f \)? (Lesson 7-2)  
   D. \( n = f - 258 \)  
   B. \( n = f - 258 \)  
   C. \( n + f = 258 \)  
   A. \( f = 2n \)

5. Express \( 5.387 \times 10^{-3} \) in standard notation. (Lesson 8-3)  
   B. \( 0.005387 \)  
   C. \( 538.7 \)  
   D. \( 5387 \)

6. The quotient \( \frac{16x^8}{8x^7} \), \( x \neq 0 \), is (Lesson 9-1)  
   C. \( 2x^2 \)  
   B. \( 8x^2 \)  
   E. \( 2x^4 \)  
   D. \( 8x^4 \)

7. What are the solutions of the equation \( 3x^2 - 48 = 0 \)? (Lesson 9-1)  
   A. \( 4, -4 \)  
   B. \( 4, \frac{1}{3} \)  
   C. \( 16, -16 \)  
   D. \( 16, \frac{1}{3} \)

8. What are the solutions of the equation \( x^2 - 3x + 8 = 6x - 6 \)? (Lesson 9-4)  
   D. \( 2, -7 \)  
   B. \( -2, -4 \)  
   C. \( 2, 4 \)  
   E. \( 2, 7 \)

9. The area of a rectangle is \( 12x^2 - 21x - 6 \). The width is \( 3x - 6 \). What is the length? (Lesson 9-5)  
   B. \( 4x + 1 \)  
   A. \( 4x - 1 \)  
   C. \( 9x + 1 \)  
   D. \( 12x - 18 \)

**Test-Taking Tip**

Questions 7 and 9 When answering a multiple-choice question, first find an answer on your own. Then, compare your answer to the answer choices given in the item. If your answer does not match any of the answer choices, check your calculations.

**Additional Practice**

See pp. 577–578 in the Chapter 9 Resource Masters for additional standardized test practice.

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**Log On for Test Practice**

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

**TestCheck and Worksheet Builder**

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
Part 2 | Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Find all values of $x$ that make the equation $6|x - 2| = 18$ true.  \(\text{Lesson 6-5} \quad 5 \text{ and } -1\)

11. Graph the inequality $x + y \leq 3$.  \(\text{Lesson 6-6} \quad \text{See margin}\)

12. A movie theater charges $7.50 for each adult ticket and $4 for each child ticket. If the theater sold a total of 145 tickets for a total of $790, how many adult tickets were sold?  \(\text{Lesson 7-2} \quad 60\)

13. Solve the following system of equations.

\[
\begin{align*}
3x + y &= 8 \\
4x - 2y &= 14
\end{align*} \quad (Lesson 7-3) \quad (3, -1)\]

14. Write $(x + 1)(x + 1)y$ as the product of two factors.  \(\text{Lesson 9-3} \quad (x + 1)(x + y)\)

15. The product of two consecutive odd integers is 195. Find the integers.  \(\text{Lesson 9-4} \quad 13 \text{ and } 15 \text{ or } -13 \text{ and } -15\)

16. Solve $2x^2 + 5x - 12 = 0$ by factoring.  \(\text{Lesson 9-5} \quad \frac{2}{3} \text{ or } -4\)

17. Factor $2x^2 + 7x + 3$.  \(\text{Lesson 9-5} \quad (2x + 1)(x + 3)\)

Part 3 | Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

\(A\) the quantity in Column A is greater,

\(B\) the quantity in Column B is greater,

\(C\) the two quantities are equal, or

\(D\) the relationship cannot be determined from the information given.

### Column A | Column B
---|---
| \(\frac{x}{3} - 27 = 39\) | \(\frac{3y}{4} - 55 = 20\) |

Part 4 | Open Ended

Record your answers on a sheet of paper. Show your work.

23. Madison is building a fenced, rectangular dog pen. The width of the pen will be 5 yards less than the length. The total area enclosed is 28 square yards.  \(\text{Lesson 9-4} \quad \text{a. Using } L \text{ to represent the length of the pen, write an expression showing the area of the pen in terms of its length.}\)

\(b. \) What is the length of the pen?

\(c. \) Madison needs to enclose the pen completely?  \(\text{a-c. See margin}\)

23a. A polynomial equation equivalent to $28 = L(L - 3)$

23b. The length is 7 yards.

\[
\begin{align*}
L^2 - 3L - 28 &= 0 \\
(L - 7)(L + 4) &= 0 \\
L &= 7
\end{align*}
\]

23c. 22 yd

Calculate $W$. Calculate the perimeter.

\[
\begin{align*}
W &= L - 3 \\
W &= 7 - 3 \text{ or } 4 \\
P &= 2L + 2W \\
P &= 2(7) + 2(4) \\
P &= 14 + 8 \text{ or } 22
\end{align*}
\]
69. Scientists listening to radio signals would suspect that a modulated signal beginning with prime numbers would indicate a message from an extraterrestrial. Answers should include the following:

- Sample answer: It is unlikely that any natural phenomenon would produce such an artificial and specifically mathematical pattern.

5.  
\[
\begin{array}{c|cccc}
2x - 5 \\
\hline
x & x & \_1 & \_1 & \_1 \\
\hline
x & x & \_1 & \_1 & \_1 \\
\hline
\end{array}
\]

6.  
\[
\begin{array}{c|cccc}
x & \_1 & \_1 & \_1 & \_1 \\
\hline
x & \_1 & \_1 & \_1 & \_1 \\
\hline
\end{array}
\]

7.  
\[
\begin{array}{c|cc}
x + 2 \\
\hline
x & x^2 & x \\
\hline
\end{array}
\]

8.  
\[
\begin{array}{c|cc}
x^2 & \_1 & \_1 \\
\hline
x^2 & \_1 & \_1 \\
\hline
\end{array}
\]

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5.  
\[
\begin{array}{c|cccc}
2x - 5 \\
\hline
x & x & \_1 & \_1 & \_1 \\
\hline
x & x & \_1 & \_1 & \_1 \\
\hline
\end{array}
\]

6.  
\[
\begin{array}{c|cccc}
x & \_1 & \_1 & \_1 & \_1 \\
\hline
x & \_1 & \_1 & \_1 & \_1 \\
\hline
\end{array}
\]

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16. \(5(x + 6y)\)  
17. \(4(4a + b)\)
18. \(a(a^4b - 1)\)  
19. \(x(x^2y^2 + 1)\)
20. \(3d(7c - 1)\)  
21. \(2h(7g - 9)\)
22. \(15ay(a - 2)\)  
23. \(8bc(c + 3)\)
24. \(4xy^2z(3x + 10yz)\)  
25. \(6abc^2(3a - 8c)\)
26. \(a(1 + ab^2 + a^2b^3)\)  
27. \(x(15xy^2 + 25y + 1)\)
28. \(4x(3ax^2 + 5bx + 8c)\)  
29. \(3pq(p^2 - 3q + 12)\)
30. \((x + 3)(x + 2)\)  
31. \((x + 7)(x + 5)\)
32. \((2x + 3)(2x + 7)\)  
33. \((3y + 2)(4y + 3)\)
34. \((3a - 4)(2a - 5)\)  
35. \((6x - 1)(3x - 5)\)
36. \((a + b)(4x + 3y)\)  
37. \((m + x)(2y + 7)\)
38. \((2x - 3)(4a - 3)\)  
39. \((2x - 3)(5x - 7y)\)

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35. \([-5, \frac{2}{5}]\)  
36. \([-\frac{4}{3}, 3]\)
37. \([-1, \frac{3}{4}]\)  
38. \(\frac{1}{3}, \frac{2}{5}\)
39. \([-\frac{5}{7}, 2]\)  
40. \([-\frac{5}{4}, \frac{7}{3}]\)
41. \([-\frac{2}{3}, 3]\)  
42. \(\frac{2}{7}, 1\)
43. \(\frac{1}{2}, 3\)  
44. \(\frac{1}{3}, \frac{9}{4}\)
45. \((-4, 12)\)  
46. \(\frac{7}{3}, \frac{5}{2}\)
47. \((-4, \frac{2}{3}]\)  
48. \(\frac{1}{2}, 3\)