# Chapter 3: Solving Linear Equations

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Pacing suggestions for the entire year can be found on pages T20–T21.
**Chapter Resource Manager**

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*Key to Abbreviations: GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

ELL Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Writing Equations

Writing equations from verbal sentences is an essential tool for solving real-world problems. Variables are used to represent unspecified amounts. There are key words to assist in writing the mathematical equations, such as equals, is, times, and, sum, difference, less, more, and so on. Use the Four-Step Problem-Solving Plan to solve problems. Always examine the solution to make sure the answer is reasonable. Translating equations to verbal sentences can help give meaning to the equations.

Solving Equations by Using Addition and Subtraction

Solving an equation means finding all the values of the variable in the equation that make the statement true. To solve an equation, isolate the variable so that it has a coefficient of 1 on one side of the equation. If a number is being added to or subtracted from the variable in the original equation, use the inverse function to isolate the variable. Use the Addition or Subtraction Property of Equality to preserve equality. These properties stress the importance of performing the same operation on each side of the equation to result in an equivalent equation.

Solving Equations by Using Multiplication and Division

You also solve equations in which the variable is multiplied or divided by a rational number by using the inverse operation. The Multiplication and Division Properties of Equality state that you can multiply or divide each side of an equation by the same number and preserve equality.

Solving Multi-Step Equations

To solve some problems, the problem-solving strategy of working backward is helpful. This strategy is used to solve multi-step equations. Working backward and using inverse operations undo the order of operations. First, like terms must be combined. Then, the opposite of the order of operations is used: the Addition or Subtraction Property of Equality is performed before the Multiplication or Division Property of Equality.
Solving Equations with the Variable on Each Side

To solve any equation, no matter how complex, the variable must always be isolated. First apply the Distributive Property if necessary. Then combine like terms on each side of the equation. Move all variable terms to one side of the equation and all numeric terms to the other side using the Addition and/or Subtraction Properties of Equality. Then apply the Multiplication or Division Property of Equality.

There is no solution if the two sides of the equation cannot be equal. This occurs when all variable terms are eliminated and the two sides of the equation are not equal numbers. If both sides are identical at any point in the solution process, then the equation is an identity. In this case, all numbers are solutions.

Ratios and Proportions

A ratio is a comparison of two numbers by division. The numbers of a ratio can be written side by side with "to" or a colon between them, or they may be written to resemble a fraction. If the two numbers of a ratio represent two different measures, such as miles and hours, the ratio is called a rate. When using a rate to make a model or drawing that is larger or smaller than the original, the rate is called a scale.

A proportion is an equation stating that two ratios are equal. One way to determine if two ratios are equivalent is to use cross products. The product of the means of a proportion equals the product of its extremes. If the products are not equal, then the ratios do not form a proportion.

If a proportion contains a variable, the proportion can be solved for that variable by setting the product of the means equal to the product of the extremes. Then solve the resulting equation using the Division Property of Equality.

Percent of Change

Percent of change is the percent amount a number increases or decreases. If the new number is greater than the original number, the percent of change is called a percent of increase. If the new number is less than the original number, it is called a percent of decrease. Percent is found by dividing a part by its corresponding whole amount. Percent of change is found by solving a proportion. The ratio of the amount of change to the original number equals the ratio of the percent to 100.

Solving Equations and Formulas

Some equations contain more than one variable. The process for solving one-step or multi-step equations is applied to solve these equations for one of the variables in terms of the other terms. Formulas are written as equations with multiple variables. They can be solved for one of the variables to make computation easier.

Weighted Averages

A weighted average is the sum of the product of the number of units in a set of data and the value per unit divided by the sum of the number of units. Two or more parts are combined into a whole in mixture problems. Weighted averages are used to solve mixture problems.

Uniform motion problems also use weighted averages. The distance formula is used to solve these problems. You solve the distance formula for the variable that the two movements have in common. A table is sometimes helpful when organizing these problems.
## Chapter 3 Daily Intervention and Assessment

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Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

### Additional Intervention Resources

- The Princeton Review's *Cracking the SAT & PSAT*
- The Princeton Review's *Cracking the ACT*
- ALEKS

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### TestCheck and Worksheet Builder

This networkable software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.
Reading and Writing in Mathematics

Glencoe Algebra 1 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

### Student Edition
- Foldables Study Organizer, p. 119
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 123, 131, 138, 145, 151, 158, 162, 168, 174)
- Reading Mathematics, p. 165
- Writing in Math questions in every lesson, pp. 126, 134, 140, 147, 154, 159, 164, 170, 177
- Reading Study Tip, pp. 121, 129, 155
- WebQuest, pp. 159, 177

### Teacher Wraparound Edition
- Foldables Study Organizer, pp. 119, 179
- Modeling activities, pp. 140, 159
- Speaking activities, pp. 126, 148, 164
- Writing activities, pp. 134, 154, 170, 177
- Differentiated Instruction, (Verbal/Linguistic), p. 121
- **ELL** Resources, pp. 118, 121, 125, 133, 139, 147, 153, 156, 158, 163, 165, 169, 176, 179

### Additional Resources
- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 3 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 3 Resource Masters*, pp. 141, 147, 153, 159, 165, 171, 177, 183, 189)
- **Vocabulary PuzzleMaker** software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- Teaching Mathematics with Foldables provides suggestions for promoting cognition and language.
- Reading and Writing in the Mathematics Classroom
- WebQuest and Project Resources
- Hot Words/Hot Topics Sections 1.3, 2.1, 2.3–2.6, 2.8, 6.1–6.5

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Solving Linear Equations

Key Vocabulary

- equivalent equations (p. 129)
- identity (p. 150)
- proportion (p. 155)
- percent of change (p. 160)
- mixture problem (p. 171)

Linear equations can be used to solve problems in every facet of life from planning a garden, to investigating trends in data, to making wise career choices. One of the most frequent uses of linear equations is solving problems involving mixtures or motion. For example, in the National Football League, a quarterback’s passing performance is rated using an equation based on a mixture, or weighted average, of five factors, including passing attempts and completions. You will learn how this rating system works in Lesson 3-9.

Vocabulary Builder

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 3 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 3 test.
This section provides a review of the basic concepts needed before beginning Chapter 3. Page references are included for additional student help. Additional review is provided in the Prerequisite Skills Workbook, pp. 9–12, 17–18, 21–24, 27–28, 41–44, 51–52, 59–60, 67–74, 77–78, 81–82.

**Prerequisite Skills** in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

### For Lesson 3-1

**Write Mathematical Expressions**

Write an algebraic expression for each verbal expression. (For review, see Lesson 1-1.)

1. five greater than half of a number $t$ $\frac{1}{2}t + 5$
2. the product of seven and $s$ divided by the product of eight and $y$ $7s \div 8y$
3. the sum of three times $a$ and the square of $b$ $3a + b^2$
4. $w$ to the fifth power decreased by 37 $w^5 - 37$
5. nine times $y$ subtracted from 95 $95 - 9y$
6. the sum of $r$ and six divided by twelve $(r + 6) + 12$

### For Lesson 3-4

**Use the Order of Operations**

Evaluate each expression. (For review, see Lesson 1-2.)

7. $3 \cdot 6 - \frac{12}{4}$ 15
8. $5(13 - 7) - 22$ 8
9. $5(7 - 2) - 3^2$ 16
10. $\frac{2 \cdot 6 - 4}{2}$ 4
11. $(25 - 4) \div (2^2 - 1)$ 12
12. $36 \div 4 - 2 + 3$ 10
13. $\frac{19 - 5}{7} + 3$ 5
14. $\frac{1}{4}(24) - \frac{1}{2}(12)$ 0

### For Lesson 3-7

**Find the Percent**

Find each percent. (For review, see pages 802 and 803.)

15. Five is what percent of 20? 25%
16. What percent of 300 is 21? 7%
17. What percent of 5 is 15? 300%
18. Twelve is what percent of 60? 20%
19. Sixteen is what percent of 10? 160%
20. What percent of 50 is 37.5? 75%

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**FOLDABLES™ Study Organizer**

Make this Foldable to help you organize information about solving linear equations. Begin with 4 sheets of plain $8\frac{1}{2}$" by 11" paper.

1. **Fold**
   - Fold in half along the width.
2. **Open and Fold Again**
   - Fold the bottom to form a pocket. Glue edges.
3. **Repeat Steps 1 and 2**
4. **Label**
   - Label each pocket. Place an index card in each pocket.

**Reading and Writing**

As you read and study the chapter, you can write notes and examples on each index card.

**Organization of Data**

Students will need 3 inch-by-5 inch index cards or sheets of notebook paper cut into fourths to use as study cards. In Lesson 3-1, have students write an equation on one side of each card and its verbal equivalent on the other side. Store these cards in the first pocket of the Foldable, labeled “3-1: Writing Equations.” With each lesson, use the study cards to take notes, solve equations, or record and define vocabulary words and concepts. There are 8 pockets. Place Lessons 3-8 and 3-9 in the same pocket.

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**5-Minute Check Transparency 3-1** Use as a quiz or review of Chapter 2.

**Mathematical Background** notes are available for this lesson on p. 118C.

**How are equations used to describe heights?**

Ask students:

- You could say that the total height of the statue, 305 feet, is equal to the sum of what two quantities? **the height of the statue and the height of the pedestal**
- Why is the height of the statue represented by a variable? **The height of the statue is not stated in the problem, so it must be represented by a variable.**
- What is another equation that could be used to represent the situation? **305 - s = 154 or 305 - 154 = s**

**WRITE EQUATIONS** When writing equations, use variables to represent the unspecified numbers or measures referred to in the sentence or problem. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the equals sign are listed below:

- **is**
- **equals**
- **is equal to**
- **is as much as**
- **is the same as**
- **is identical to**

**Example 1 Translate Sentences into Equations**

Translate each sentence into an equation.

a. Five times the number $a$ is equal to three times the sum of $b$ and $c$.

$\frac{5}{3} \times a = \frac{3}{3} \times (b + c)$

The equation is $5a = 3(b + c)$.

b. Nine times $y$ subtracted from 95 equals 37.

$95 - 9y = 37$

The equation is $95 - 9y = 37$. 

**Resource Manager**

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| Parent and Student Study Guide Workbook, p. 19 |
| Teaching Algebra with Manipulatives Masters, p. 56 |

| Transparencies |
| 5-Minute Check Transparency 3-1 |
| Answer Key Transparencies |

| Technology |
| Interactive Chalkboard |
Lesson 3-1
Writing Equations

Using the four-step problem-solving plan can help you solve any word problem.

Key Concept

Four-Step Problem-Solving Plan

Step 1 Explore the problem.
Step 2 Plan the solution.
Step 3 Solve the problem.
Step 4 Examine the solution.

Each step of the plan is important.

Step 1 Explore the Problem
To solve a verbal problem, first read the problem carefully and explore what the problem is about.
• Identify what information is given.
• Identify what you are asked to find.

Step 2 Plan the Solution
One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called defining a variable. Then use the variable to write expressions for the other unspecified numbers in the problem. You will learn to use other strategies throughout this book.

Step 3 Solve the Problem
Use the strategy you chose in Step 2 to solve the problem.

Step 4 Examine the Solution
Check your answer in the context of the original problem.
• Does your answer make sense?
• Does it fit the information in the problem?

Example 2 Use the Four-Step Plan

ICE CREAM Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?

Explore You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

Plan Write an equation to represent the situation. Let \( d \) represent the number of days needed to produce the ice cream.

\[
\frac{2,000,000}{20} \times \frac{\text{days}}{d} = \frac{40,000,000}{40,000,000}
\]

Solve \( 2,000,000d = 40,000,000 \)

\[
20 = \frac{40,000,000}{d}
\]

Examine If 2,000,000 gallons of ice cream are produced in one day, 2,000,000 \( \times \) 20 or 40,000,000 gallons are produced in 20 days. The answer makes sense.

More About . . .

Ice Cream
The first ice cream plant was established in 1851 by Jacob Fussell. Today, 2,000,000 gallons of ice cream are produced in the United States each day. Source: World Book Encyclopedia

www.algebra1.com/extra_examples

Lesson 3-1 Writing Equations 121

WRITE EQUATIONS

In-Class Examples

Reading Tip Remind students that the language of math sentences is not always as obvious as in the examples.

1 Translate each sentence into an equation.

a. A number \( b \) divided by three is equal to six less than \( c \).

\[
\frac{b}{3} = c - 6
\]

b. Fifteen more than \( z \) times 6 is \( y \) times 2 minus eleven.

\[
15 + 6z = 2y - 11
\]

2 JELLYBEANS A popular jellybean manufacturer produces 1,250,000 jellybeans per hour. How many hours does it take them to produce 10,000,000 jellybeans? 8 hours

Example

Translate each sentence into an equation.

a. A number \( b \) divided by three is equal to six less than \( c \).

\[
\frac{b}{3} = c - 6
\]

b. Fifteen more than \( z \) times 6 is \( y \) times 2 minus eleven.

\[
15 + 6z = 2y - 11
\]

Verbal/Linguistic Some students will likely translate sentences into equations easily. Pair those students with others who are having trouble translating sentences. Have the pairs work several example problems.

Interactive Chalkboard PowerPoint® Presentations
This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:
• Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
• Additional, Your Turn exercises for each example
• The 5-Minute Check Transparencies
• Hot links to Glencoe Online Study Tools
In-Class Example

3 Teaching Tip

Some students may have the formula for the perimeter of a rectangle memorized from previous mathematics courses. Have these students work backward from their memorized formula to confirm that it is a correct translation of the given sentence.

Translate the sentence into a formula.
The perimeter of a square equals four times the length of the side. \( P = 4s \)

WRITE VERBAL SENTENCES

In-Class Example

Reading Tip

Remind students that there is often more than one way to translate an equation into a verbal sentence. For example, \( 3m + 5 = 14 \) could also be translated as, “The sum of three times \( m \) and 5 is 14.”

4 Translate each equation into a verbal sentence.

- \( 12 - 2x = -5 \) Twelve minus two times \( x \) equals negative five.
- \( a^2 + 3b = \frac{c}{6} \) \( a \) squared plus three times \( b \) equals \( c \) divided by 6.

Answers

3. Sample answer: After sixteen people joined the drama club, there were 30 members. How many members did the club have before the new members?

10. Sample answer: The original cost of a suit is \( c \). After a $25 discount, the suit costs $150. What is the original cost of the suit?

Example 3 Write a Formula

Translate the sentence into a formula.
The perimeter of a rectangle equals two times the length plus two times the width.

Words Perimeter equals two times the length plus two times the width.

Variables Let \( P = \) perimeter, \( \ell = \) length, and \( w = \) width.

Formula

\[
P = 2\ell + 2w
\]

The formula for the perimeter of a rectangle is \( P = 2\ell + 2w \).

Example 4 Translate Equations into Sentences

Translate each equation into a verbal sentence.

- \( 3m + 5 = 14 \) Three times \( m \) plus five equals fourteen.

Study Tip

Look Back

To review translating algebraic expressions to verbal expressions, see Lesson 1-1.

Algebra Activity

Materials: scissors, rectangular box

- Suggest that in addition to marking the box sides with length, width, or height, students should also label the sides as front, back, side 1, side 2, top, and bottom.
- By cutting the sides of the box into individual rectangles, students can more easily see all six components (sides) that make up the surface area of the box.
Example 5 Write a Problem

Write a problem based on the given information.

\[ a = \text{Rafael’s age} \quad a + 5 = \text{Tierra’s age} \quad a + 2(a + 5) = 46 \]
You know that \( a \) represents Rafael’s age and \( a + 5 \) represents Tierra’s age. The equation adds \( a \) plus twice \((a + 5)\) to get 46. A sample problem is given below.

Tierra is 5 years older than Rafael. The sum of Rafael’s age and twice Tierra’s age equals 46. How old is Rafael?

Check for Understanding

Concept Check

1. List the four steps used in solving problems.
2. Analyze the following problem.  
   \[ 2b. \quad $300; $600 \]
   Misae has $1900 in the bank. She wishes to increase her account to a total of $3500 by depositing $30 per week from her paycheck. Will she reach her savings goal in one year?
   a. How much money did Misae have in her account at the beginning? $1900
   b. How much money will Misae add to her account in 10 weeks? in 20 weeks? $30 \times 10 = 300 \quad 30 \times 20 = 600$
   c. Write an expression representing the amount added to the account after \( w \) weeks have passed. \( 30w \)
   d. What is the answer to the question? Explain. Yes.

3. OPEN ENDED Write a problem that can be answered by solving \( x + 16 = 30 \). See margin.

Guided Practice

GUIDED PRACTICE KEY

Exercises | Examples
---|---
4, 5 | 1
6, 7 | 3
8, 9 | 4
10 | 5
11, 12 | 2

4. Two times a number t decreased by eight equals seventy. \( 2t - 8 = 70 \)
5. Five times the sum of \( m \) and \( n \) is the same as seven times \( n \). \( 5(m + n) = 7n \)

6. The area \( A \) of a triangle equals one half times the base \( b \) times the height \( h \).
7. The circumference \( C \) of a circle equals the product of two, \( \pi \), and the radius \( r \).
8. 14 plus \( d \) equals 6 times \( d \). \( 14 + d = 6d \)
9. \( \frac{1}{3} b - \frac{3}{4} = 2a \)
   \( \frac{1}{3} \) of \( b \) minus \( \frac{3}{4} \) equals 2 times \( a \).
10. Write a problem based on the given information. See margin.
   \[ c = \text{cost of a suit} \quad c - 25 = 150 \]

Application WRESTLING For Exercises 11 and 12, use the following information.
Darius is training to prepare for wrestling season. He weighs 155 pounds now. He wants to gain weight so that he starts the season weighing 160 pounds.
11. If \( g \) represents the number of pounds he wants to gain, write an equation to represent the situation. \( 155 + g = 160 \)
12. How many pounds does Darius need to gain to reach his goal? \( 5 \) lb
Translate each sentence into an equation.

13. Two hundred minus three times \( x \) is equal to nine. \( 200 - 3x = 9 \)

14. The sum of twice \( r \) and three times \( s \) is identical to thirteen. \( 2r + 3s = 13 \)

15. The sum of one-third \( q \) and 25 is as much as twice \( q \). \( \frac{1}{3}q + 25 = 2q \)

16. The square of \( m \) minus the cube of \( n \) is sixteen. \( m^2 - n^3 = 16 \)

17. Two times the sum of \( v \) and \( w \) is equal to two times \( z \). \( 2(v + w) = 2z \)

18. Half of the sum of nine and \( p \) is the same as \( p \) minus three. \( \frac{1}{2}(9 + p) = p - 3 \)

19. The number \( g \) divided by the number \( h \) is the same as seven more than twice the sum of \( g \) and \( h \). \( \frac{g}{h} = 2(g + h) + 7 \)

20. Five-ninths the square of the sum of \( a \), \( b \), and \( c \) equals the sum of the square of \( a \) and the square of \( c \). \( \frac{5}{9}(a + b + c)^2 = a^2 + c^2 \)

21. **GEOGRAPHY** The Pacific Ocean covers about 46% of Earth. If \( P \) represents the area of the Pacific Ocean and \( E \) represents the area of Earth, write an equation for this situation. \( 0.46E = P \)

22. **GARDENING** Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs \$1.75 per yard. She buys \( f \) yards of fencing and pays \$3.50 in tax. If the total cost of the fencing is \$73.50, write an equation to represent the situation. \( 1.75f + 3.50 = 73.50 \)

Translate each sentence into a formula.

23. The area \( A \) of a parallelogram is the base \( b \) times the height \( h \). \( A = bh \)

24. The volume \( V \) of a pyramid is one-third times the product of the area of the base \( B \) and its height \( h \). \( V = \frac{1}{3} Bh \)

25. The perimeter \( P \) of a parallelogram is twice the sum of the lengths of the two adjacent sides, \( a \) and \( b \). \( P = 2(a + b) \)

26. The volume \( V \) of a cylinder equals the product of \( \pi \), the square of the radius \( r \) of the base, and the height. \( V = \pi r^2h \)

27. In a right triangle, the square of the measure of the hypotenuse \( c \) is equal to the sum of the squares of the measures of the legs, \( a \) and \( b \). \( c^2 = a^2 + b^2 \)

28. The temperature in degrees Fahrenheit \( F \) is the same as nine-fifths of the degrees Celsius \( C \) plus thirty-two. \( F = \frac{9}{5}C + 32 \)
Translate each equation into a verbal sentence.
29. \( d - 14 = 5 \)  
30. \( 2f + 6 = 19 \)  
31. \( k^2 + 17 = 53 - j \)  
32. \( 2a = 7a - b \)  
33. \( \frac{3}{4} p + \frac{1}{2} = p \)  
34. \( \frac{5}{2} w = 2w + 3 \)  

GEOMETRY
If \( a \) and \( b \) represent the lengths of the bases of a trapezoid and \( h \) represents its height, then the formula for the area \( A \) of the trapezoid is \( A = \frac{1}{2}(a + b) \). Write the formula in words.

SCIENCE
If \( r \) represents rate, \( t \) represents time, and \( d \) represents distance, then \( rt = d \). Write the formula in words. Rate times time equals distance.

Write a problem based on the given information. 39–40. See margin.
39. \( y = \) Yoland’s height in inches  
40. \( p = \) price of a new backpack

GEOMETRY
For Exercises 41 and 42, use the following information. The volume \( V \) of a cone equals one-third times the product of \( \pi \), the square of the radius \( r \) of the base, and the height \( h \).
41. Write the formula for the volume of a cone. \( V = \frac{1}{3} \pi r^2h \)
42. Find the volume of a cone if \( r = 10 \) centimeters and \( h = 30 \) centimeters. About 3142 cm\(^3\)

GEOMETRY
For Exercises 43 and 44, use the following information. The volume \( V \) of a sphere is four-thirds times \( \pi \) times the radius \( r \) of the sphere cubed.
43. Write a formula for the volume of a sphere. \( V = \frac{4}{3} \pi r^3 \)
44. Find the volume of a sphere if \( r = 4 \) inches. About 268 in\(^3\)

LITERATURE
For Exercises 45–47, use the following information. Edgar Rice Burroughs is the author of the Tarzan of the Apes stories. He published his first Tarzan story in 1912. Some years later, in the town of Southern California where he lived was named Tarzana. 46. 1912 + \( y \) = 1928
47. Let \( y \) represent the year numbers after 1912 that the town was named Tarzana. Write an expression for the year the town was named. 1912 + \( y \)
48. The town was named in 1928. Write an equation to represent the situation.
49. Use what you know about numbers to determine the number of years between the first Tarzan story and the naming of the town. 18 yr

TELEVISION
For Exercises 48–51, use the following information. During a highly rated one-hour television program, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion.
50. If \( a \) represents the time spent on advertising, write an expression for the entertainment portion. \( 4a + 15 \)
51. Use your equation and the guess-and-check strategy to determine the number of minutes spent on advertising. Choose different values of \( a \) and evaluate to find the solution. 9 min

Time the entertainment and advertising portions of a one-hour television program you like to watch. Describe what you found. Are the results of this problem similar to your findings? See students’ work.

www.algebra1.com/self_check_quiz

Lesson 3-1 Writing Equations 125
Open-Ended Assessment

Speaking  Translating sentences into equations and vice versa presents an excellent opportunity for students to practice their speaking skills. Ask volunteers to translate sentences into equations and equations into sentences aloud for the entire class to hear.

Getting Ready for Lesson 3-2

PREREQUISITE SKILL  In Lesson 3-2, students will learn how to solve equations using addition and subtraction. In addition to solving equations involving integers, students solve equations involving decimals and fractions. Use Exercises 69–76 to determine your students’ familiarity with finding sums and differences of decimals and fractions.

Answer

53. Equations can be used to describe the relationships of the heights of various parts of a structure. Answers should include the following.
   • The equation representing the Sears Tower is $1454 + a = 1707$.

Maintain Your Skills

Mixed Review

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if the result is not a whole number or a simple fraction. (Lesson 2-7)

56. $\sqrt{8100}$ 90 57. $-\sqrt{\frac{25}{36}} - \frac{5}{6}$
58. $\sqrt{90}$ 9.49 59. $-\sqrt{55}$ 7.42

Find the probability of each outcome if a die is rolled. (Lesson 2-6)

60. a 6 $\frac{1}{6}$ 61. an even number $\frac{1}{2}$ 62. a number greater than 2 $\frac{2}{3}$

Simplify each expression. (Lesson 1-5)

63. $12d + 3 - 4d$ 8d + 3 64. $7t^2 + t + 8t$ 7t^2 + 9t
65. $3(a + 2b) + 5a$ 8a + 6b

Evaluate each expression. (Lesson 1-2)

66. $5(8 - 3) + 7 \cdot 2$ 39 67. $6(4^2 + 2^2)$ 408 68. $7(0.2 + 0.5) - 0.6$ 4.3

Getting Ready for the Next Lesson

PREREQUISITE SKILL  Find each sum or difference. (To review operations with fractions, see pages 798 and 799.)

69. $5.67 + 3.7$ 9.37 70. $0.57 + 2.8$ 3.37 71. $5.28 - 3.4$ 1.88 72. $9 - 7.35$ 1.65
73. $\frac{2}{3} + \frac{1}{5}$ 13 74. $\frac{1}{6} + \frac{5}{3}$ 6 75. $\frac{7}{9} - \frac{2}{3}$ 1 76. $\frac{3}{4} - \frac{1}{6}$ 7 12
Solving Addition and Subtraction Equations

You can use algebra tiles to solve equations. To solve an equation means to find the value of the variable that makes the equation true. After you model the equation, the goal is to get the x tile by itself on one side of the mat using the rules stated below.

### Rules for Equation Models

<table>
<thead>
<tr>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>You can remove or add the same number of identical algebra tiles to each side of the mat without changing the equation.</td>
<td><img src="image1" alt="Example" /></td>
</tr>
<tr>
<td>One positive tile and one negative tile of the same unit are a zero pair. Since 1 + (−1) = 0, you can remove or add zero pairs to the equation mat without changing the equation.</td>
<td><img src="image2" alt="Example" /></td>
</tr>
</tbody>
</table>

### Use an equation model to solve \( x - 3 = 2 \).  

**Step 1** Model the equation.

![Modeling the equation](image3)

\[
x - 3 = 2
\]

\[
x - 3 + 3 = 2 + 3
\]

Place 1 x tile and 3 negative 1 tiles on one side of the mat. Place 2 positive 1 tiles on the other side of the mat. Then add 3 positive 1 tiles to each side.

**Step 2** Isolate the x term.

![Isolating the x term](image4)

\[
x = 5
\]

Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is \( x = 5 \).

### Model and Analyze

Use algebra tiles to solve each equation.

1. \( x + 5 = 7 \)
2. \( x + (−2) = 28 - 6 \)
3. \( x + 4 = 27 - 11 \)
4. \( x + (−3) = 4 \)
5. \( x + 3 = −4 − 7 \)
6. \( x + 7 = 2 − 5 \)

### Make a Conjecture

7. If \( a = b \), what can you say about \( a + c \) and \( b + c \)? \( a + c = b + c \)
8. If \( a = b \), what can you say about \( a − c \) and \( b − c \)? \( a − c = b − c \)

### Teaching Algebra with Manipulatives

- pp. 10–11 (masters for algebra tiles)
- p. 16 (master for equation mat)
- p. 59 (student recording sheet)

### Glencoe Mathematics Classroom Manipulative Kit

- algebra tiles
- equation mat

### Resource Manager

**Teaching Algebra with Manipulatives**

- pp. 10–11 (masters for algebra tiles)
- p. 16 (master for equation mat)
- p. 59 (student recording sheet)

**Glencoe Mathematics Classroom Manipulative Kit**

- algebra tiles
- equation mat

### Study Notebook

You may wish to have students summarize this activity and what they learned from it.

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**Algebra Activity**

**Solving Addition and Subtraction Equations**

**Objective** Use equation mats and algebra tiles to model solving equations.

**Materials**

- equation mats
- algebra tiles

**Teach**

- Remind students that when they are modeling subtraction with algebra tiles, they must add negative tiles.
- Explain to students that they can remove or add the same number of identical algebra tiles to each side of the mat or they can remove or add a zero pair to one side of the mat.
- There are not enough negative 1 tiles on the right side of the mat, so that 3 negative 1 tiles can be removed from each side as in Example 1. In this case, add 3 positive 1 tiles to both sides to create three zero-pairs on the left side.

**Assess**

After Exercises 1–6, students need to recognize that zero pairs must be formed.  
Exercises 7–8 provide a symbolic representation for the activity.
Solving Equations by Using Addition and Subtraction

What You’ll Learn

• Solve equations by using addition.
• Solve equations by using subtraction.

Vocabulary

• equivalent equation
• solve an equation

How can equations be used to compare data?

The graph shows some of the fastest-growing occupations from 1992 to 2005.

Selected Fastest-Growing Occupations

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Percent of growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical therapist</td>
<td>88%</td>
</tr>
<tr>
<td>Paralegals</td>
<td>86%</td>
</tr>
<tr>
<td>Detective</td>
<td>70%</td>
</tr>
<tr>
<td>Correction officer</td>
<td>70%</td>
</tr>
<tr>
<td>Travel agent</td>
<td>66%</td>
</tr>
</tbody>
</table>

The difference between the percent of growth for medical assistants and the percent of growth for travel agents in these years is 5%. An equation can be used to find the percent of growth expected for medical assistants. If \( m \) is the percent of growth for medical assistants, then \( m - 66 = 5 \). You can use a property of equality to find the value of \( m \).

SOLVE USING ADDITION

Suppose your school’s boys’ soccer team has 15 members and the girls’ soccer team has 15 members. If each team adds 3 new players, the number of members on the boys’ and girls’ teams would still be equal.

\[
\begin{align*}
15 & = 15 & \text{Each team has 15 members before adding the new players.} \\
15 + 3 & = 15 + 3 & \text{Each team adds 3 new members.} \\
18 & = 18 & \text{Each team has 18 members after adding the new members.}
\end{align*}
\]

This example illustrates the Addition Property of Equality.

Key Concept

Addition Property of Equality

- **Words** If the same number is added to each side of an equation, the resulting equation is true.
- **Symbols** For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( a + c = b + c \).
- **Examples**

\[
\begin{align*}
7 & = 7 & 14 & = 14 \\
7 + 3 & = 7 + 3 & 14 + (-6) & = 14 + (-6) \\
10 & = 10 & 8 & = 8
\end{align*}
\]

Resource Manager

Workbook and Reproducible Masters

- **Chapter 3 Resource Masters**
  - Study Guide and Intervention, pp. 143–144
  - Skills Practice, p. 145
  - Practice, p. 146
  - Reading to Learn Mathematics, p. 147
  - Enrichment, p. 148

- **Parent and Student Study Guide Workbook**, p. 20
- **Prerequisite Skills Workbook**, pp. 21–22, 59–60

Transparencies

- 5-Minute Check Transparency 3-2
- Answer Key Transparencies

Technology

Interactive Chalkboard
If the same number is added to each side of an equation, then the result is an equivalent equation. **Equivalent equations** have the same solution.

- \( t + 3 = 5 \)  
  The solution of this equation is 2.
- \( t + 3 + 2 = 5 + 2 \)  
  Using the Addition Property of Equality, add 2 to each side.
- \( t + 5 = 7 \)  
  The solution of this equation is also 2.

To **solve an equation** means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable having a coefficient of 1 on one side of the equation. You can sometimes do this by using the Addition Property of Equality.

**Example 1** Solve by Adding a Positive Number

Solve \( m - 48 = 29 \). Then check your solution.

\[
\begin{align*}
  m - 48 &= 29 & \text{Original equation} \\
  m - 48 + 48 &= 29 + 48 & \text{Add 48 to each side.} \\
  m &= 77 & -48 + 48 = 0 \text{ and } 29 + 48 = 77
\end{align*}
\]

To check that 77 is the solution, substitute 77 for \( m \) in the original equation.

<table>
<thead>
<tr>
<th>CHECK</th>
<th>( m - 48 = 29 )</th>
<th>Original equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>77 - 48 ( \neq ) 29</td>
<td>Substitute 77 for ( m ).</td>
</tr>
<tr>
<td></td>
<td>29 = 29 ( \checkmark )</td>
<td>Subtract.</td>
</tr>
</tbody>
</table>

The solution is 77.

**Example 2** Solve by Adding a Negative Number

Solve \( 21 + q = -18 \). Then check your solution.

\[
\begin{align*}
  21 + q &= -18 & \text{Original equation} \\
  21 + q + (-21) &= -18 + (-21) & \text{Add } -21 \text{ to each side.} \\
  q &= -39 & \text{Add} \\
  \end{align*}
\]

<table>
<thead>
<tr>
<th>CHECK</th>
<th>( 21 + q = -18 )</th>
<th>Original equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21 + (-39) ( \neq ) -18</td>
<td>Substitute -39 for ( q ).</td>
</tr>
<tr>
<td></td>
<td>-18 = -18 ( \checkmark )</td>
<td>Add.</td>
</tr>
</tbody>
</table>

The solution is -39.

**SOLVE USING SUBTRACTION** Similar to the Addition Property of Equality, there is a **Subtraction Property of Equality** that may be used to solve equations.

**Key Concept**

Subtraction Property of Equality

- **Words**  
  If the same number is subtracted from each side of an equation, the resulting equation is true.
- **Symbols**  
  For any numbers \( a, b, \) and \( c \), if \( a = b \), then \( a - c = b - c \).
- **Examples**  
  
  \[
  \begin{align*}
  17 &= 17 & \quad 3 = 3 \\
  17 - 9 &= 17 - 9 & \quad 3 - 8 = 3 - 8 \\
  8 &= 8 & \quad -5 = -5
  \end{align*}
  \]

**Unlocking Misconceptions**

**Isolating Variables** Explain to students that when isolating a variable, it does not matter whether the variable ends up on the left or right side of an equation. For example, the solution of \( 8 = 15 + z \) is still \( -7 \), even though the final step may be \( -7 = z \).
In-Class Examples

3 Solve \( c + 102 = 36 \). Then check your solution. \( c = -66 \)

4 Solve \( y + \frac{4}{5} = \frac{2}{3} \) in two ways.
\[ y = -\frac{2}{15} \]

Teaching Tip Tell students that they may use whichever method is most comfortable for them when solving equations.

Concept Check

Solving Equations Ask students to describe the types of equations they would solve using the Subtraction Property and what types they would solve using the Addition Property. Look for logical choices that make computation simpler.

In-Class Example

Teaching Tip Students may try to skip a step and solve the problem without first writing an equation. Tell students that they will make fewer mistakes in solving equations if they first translate the sentence and write down the equation, before trying to solve it.

5 Write an equation for the problem. Then solve the equation and check your solution.
Fourteen more than a number is equal to twenty-seven. Find the number.
\[ 14 + n = 27 \]
\[ n = 13 \]

Example 3 Solve by Subtracting

Solve \( 142 + d = 97 \). Then check your solution.
\[ 142 + d = 97 \] Original equation
\[ 142 + d - 142 = 97 - 142 \] Subtract 142 from each side.
\[ d = -45 \]
\[ 142 - 142 = 0 \text{ and } 97 - 142 = -45 \]
CHECK \[ 142 + d = 97 \] Original equation
\[ 142 + (-45) = 97 \] Substitute -45 for \( d \).
\[ 97 = 97 \checkmark \] Add.

The solution is -45.

Remember that subtracting a number is the same as adding its inverse.

Example 4 Solve by Adding or Subtracting

Solve \( g + \frac{3}{4} = -\frac{1}{8} \) in two ways.
Method 1 Use the Subtraction Property of Equality.
\[ g + \frac{3}{4} = -\frac{1}{8} \] Original equation
\[ g + \frac{3}{4} - \frac{3}{4} = -\frac{1}{8} - \frac{3}{4} \] Subtract \( \frac{3}{4} \) from each side.
\[ g = -\frac{7}{8} \]
\[ \frac{3}{4} - \frac{3}{4} = 0 \text{ and } -\frac{1}{8} - \frac{3}{4} = -\frac{1}{8} - \frac{6}{8} = -\frac{7}{8} \]
The solution is \( -\frac{7}{8} \).

Method 2 Use the Addition Property of Equality.
\[ g + \frac{3}{4} = -\frac{1}{8} \] Original equation
\[ g + \frac{3}{4} + \left( -\frac{3}{4} \right) = -\frac{1}{8} + \left( -\frac{3}{4} \right) \] Add \( -\frac{3}{4} \) to each side.
\[ g = -\frac{7}{8} \]
\[ \frac{3}{4} + \left( -\frac{3}{4} \right) = 0 \text{ and } -\frac{1}{8} + \left( -\frac{3}{4} \right) = -\frac{1}{8} + \left( -\frac{6}{8} \right) \text{ or } -\frac{7}{8} \]
The solution is \( -\frac{7}{8} \).

Example 5 Write and Solve an Equation

Write an equation for the problem. Then solve the equation and check your solution.
A number increased by 5 is equal to 42. Find the number.
\[ \frac{\text{A number}}{\text{increased by 5}} = \frac{42}{5} \]
\[ n + 5 = 42 \] Original equation
\[ n + 5 - 5 = 42 - 5 \] Subtract 5 from each side.
\[ n = 37 \]
\[ 5 - 5 = 0 \text{ and } 42 - 5 = 37 \]
CHECK \[ n + 5 = 42 \] Original equation
\[ 37 + 5 = 42 \] Substitute 37 for \( n \).
\[ 42 = 42 \checkmark \]
The solution is 37.

Differentiated Instruction

Visual/Spatial Students will most easily grasp the concept of solving equations by addition or subtraction if they physically observe adding or removing objects from both sides of the equals sign. Use the procedures from the Algebra Activity on page 127 to solve simple equations.
In the fourteenth century, the part of the Great Wall of China that was built during Qui Shi Huangdi’s time was repaired, and the wall was extended. When the wall was completed, it was 2500 miles long. How much of the wall was added during the 1300s?

**Words**  
The original length plus the additional length equals 2500.

**Variable**  
Let \( a \) = the additional length.

\[
\begin{align*}
\text{The original length} & \quad \text{plus} \quad \text{the additional length} \quad \text{equals} \quad 2500. \\
1000 & \quad + \quad a \quad = \quad 2500
\end{align*}
\]

**Equation**  
Original equation  
1000 + \( a \) = 2500

Subtract 1000 from each side.  
\[
\frac{1000 - 1000}{1000 - 1000} \quad 1000 - 1000 = 0 \quad \text{and} \quad 2500 - 1000 = 1500.
\]

The Great Wall of China was extended 1500 miles in the 1300s.

---

**Example 6** Write an Equation to Solve a Problem

**HISTORY** Refer to the information at the right.

The Washington Monument in Washington, D.C., was built in two phases. During the first phase, from 1848–1854, the monument was built to a height of 152 feet. From 1854 until 1878, no work was done. Then from 1878 to 1888, the additional construction resulted in its final height of 555 feet. How much of the monument was added during the second construction phase? Write an equation to solve the problem.

\[152 + a = 555; \quad a = 403 \text{ ft}\]
Solve each equation. Then check your solution. 29. \(-2.58\)

15. \(v - 9 = 14\)  

16. \(s - 19 = -34\)  

17. \(g + 5 = 33\)

18. \(18 + z = 44\)  

19. \(a - 55 = -17\)  

20. \(t - 72 = -44\)

21. \(-18 = -61 + d\)  

22. \(-25 = -150 + q\)  

23. \(r - (-19) = -77\)

24. \(b - (-65) = 15\)  

25. \(18 - (-f) = 91\)  

26. \(125 - (-u) = 88\)

27. \(-2.56 + c = 0.89\)  

28. \(k + 0.6 = -3.84\)  

29. \(-6 = m + (-3.42)\)

30. \(6.2 = -4.83 + y\)  

31. \(t - 8.5 = 7.15\)  

32. \(q - 2.78 = 4.2\)

33. \(x - \frac{3}{4} = \frac{5}{6}\)  

34. \(a - \frac{3}{5} = \frac{7}{10}\)  

35. \(-\frac{1}{2} + p = \frac{5}{8}\)

36. \(\frac{2}{3} + r = -\frac{4}{9}\)  

37. \(\frac{2}{3} = \frac{v}{4} + \frac{2}{5}\)  

38. \(\frac{2}{5} = w + \frac{3}{4}\)

39. If \(x - 7 = 14\), what is the value of \(x - 2\)?  

40. If \(t + 8 = -12\), what is the value of \(t + 1\)?

41. Write an equation you could use to solve for \(x\) and then solve for \(x\).

42. Write an equation you could use to solve for \(y\) and then solve for \(y\).  

Write an equation for each problem. Then solve the equation and use the rectangle at the right.  

43. Eighteen subtracted from a number equals 21. Find the number.

44. What number decreased by 77 equals \(-18\)? \(n - 77 = -18; 59\)

45. A number increased by \(-16\) is \(-21\). Find the number. \(n + (-16) = -21; -5\)

46. The sum of a number and \(-43\) is 102. What is the number? \(n + (-43) = 102; 145\)

47. What number minus one-half is equal to negative three-fourths?  

48. The sum of 19 and 42 and a number is equal to 87. What is the number? \(19 + 42 + n = 87; 26\)

49. Determine whether \(x + x = x\) is sometimes, always, or never true. Explain your reasoning. Sometimes; if \(x = 0\), \(x + x = x\) is true.

50. Determine whether \(x + 0 = x\) is sometimes, always, or never true. Explain your reasoning. Always; any number plus 0 is always the number.

51. Write an addition equation to represent the situation. \(\ell + 10 = 34\)

52. How many miles does a luxury car travel on a gallon of gasoline? \(24\) mi

53. A subcompact car with a 3-cylinder engine goes 13 miles more than a luxury car on one gallon of gas. How far does a subcompact car travel on a gallon of gasoline? \(37\) mi

54. How many more miles does a subcompact travel on a gallon of gasoline than a midsize car? \(3\) mi

55. Estimate how many miles a full-size car with a 6-cylinder engine goes on one gallon of gasoline. Explain your reasoning. Sample answer: 29 mi; 29 is the average of 24 (for the 8-cylinder engine) and 34 (for the 4-cylinder engine).
**HISTORY** For Exercises 56 and 57, use the following information.
Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased.

56. Write an addition equation to represent the situation. \(450 + d = 481\)
57. What was the decrease in the height of the pyramid? 31 ft

**LIBRARIES** For Exercises 58–61, use the graph at the right to write an equation for each situation. Then solve the equation.

58. How many more volumes does the Library of Congress have than the Harvard University Library?
59. How many more volumes does the Harvard University Library have than the New York Public Library?
60. How many more volumes does the Library of Congress have than the New York Public Library?
61. What is the total number of volumes in the three largest U.S. libraries?

**ANIMALS** For Exercises 62–64, use the information below to write an equation for each situation. Then solve the equation.

Wildlife authorities monitor the population of animals in various regions. One year’s deer population in Dauphin County, Pennsylvania, is shown in the graph below.

**Enrichment, p. 148**

**Counting-Off Puzzles** Solve each puzzle.

- **Puzzle A:** Twenty people are standing in a circle. Starting with person 1, they count off from 1 to 2 and then start over with 1. Each person who says “1” stops at the circle. Those who say the last person call “1” number 15.

- **Puzzle B:** Forty people stand in a circle. They count off in the same order as before. Which two people are the last to count? 13th and 24th person?

**Solve Each Puzzle:**

1. Solve the Subtraction Property of Equality to solve an equation. Explain why it would also be possible to use the Addition Property of Equality to solve the equation. Subtracting one number from another gives the same result as adding the opposite of the number that was subtracted.

**Helping You Remember**

- Explain how you decide whether to use the Addition Property or the Subtraction Property of Equality to solve an equation.

**Reading to Learn Mathematics**

**ELL**

**Pre-Activity** How can equations be used to compare data?

- Read the introduction to Lessons 3-3 at the top of page 138 in your textbook. In equation \(n - 60 = 5\), the number 1 represents the difference between the percent of growth for medical assistants and the percent of growth for travel agents, and the number 60 represents the end of the growth for travel agents.
Open-Ended Assessment

Writing Have students pick an example problem from the exercises of this lesson, and explain in writing how to solve the problem using addition or subtraction.

Getting Ready for Lesson 3-3

PREREQUISITE SKILL Students will learn how to solve equations using multiplication and division in Lesson 3-3. In addition to integers, they will solve equations involving decimals and fractions. Use Exercises 82–89 to determine your students’ familiarity with finding products and quotients of decimals and fractions.

Answers

66. Equations can be used to describe the relationships of growth and decline in job opportunities. Answers should include the following.

- To solve the equation, add 66 to each side. The solution is m = 71.
- An example such as “The percent increase in growth for paralegals is 16 more than the percent increase in growth for detectives. If the growth rate for paralegals is 86%, what is the growth rate for detectives? d + 16 = 86; 70%”

67. Which equation is not equivalent to b − 15 = 32? C
A b + 5 = 52
B b − 20 = 27
C b = 47

68. What is the solution of x − 167 = −52? A
A 115
B −115
C 219

Maintain Your Skills

Mixed Review

GEOMETRY For Exercises 69 and 70, use the following information. The area of a circle is the product of π times the radius r squared. (Lesson 3-1)

69. Write the formula for the area of the circle. A = \pi r^2
70. If a circle has a radius of 16 inches, find its area. About 804 in²

Replace each ⋅ with >, <, or = to make the sentence true. (Lesson 2-7)
71. \(\frac{1}{2} \cdot \sqrt{2} < \) 72. \(\frac{3}{4} \cdot \frac{2}{3} > \) 73. 0.375 ⋅ \(\frac{3}{8} = \)

Use each set of data to make a stem-and-leaf plot. (Lesson 2-5)
74. 54, 52, 43, 41, 40, 36, 35, 31, 32, 34, 42, 56
75. 2.3, 1.4, 1.7, 1.2, 2.6, 0.8, 0.5, 2.8, 4.1, 2.9, 4.5, 1.1

Identify the hypothesis and conclusion of each statement. (Lesson 1-7)
76. For y = 2, 4y − 6 = 2. H: if y = 2; C: 4y − 6 = 2
77. There is a science quiz every Friday. H: if it is Friday; C: there will be a science quiz

Evaluate each expression. Name the property used in each step. (Lesson 1-4)
78. \(4(16 + 4^2) \) 79. \((2^5 - 5^2) + (4^2 - 2^4) \) 78–79. See margin for properties used in each step.

Find the solution set for each inequality, given the replacement set. (Lesson 1-3)
80. 3x + 2 > 2; \{0, 1, 2\} \{1, 2\}
81. \(2y^2 - 1 > 0; \{1, 3, 5\} \{1, 3, 5\}

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each product or quotient. (To review operations with fractions, see pages 800 and 801.)
82. \(6.5 \times 2.8 \) 83. \(70.3 \times 0.15 \) 84. \(17.8 \div 2.5 \) 85. \(0.33 \div 1.5 \) 86. \(\frac{2}{3} \times \frac{5}{8} \) 87. \(\frac{5}{9} \times \frac{3}{10} \) 88. \(\frac{1}{2} \div \frac{2}{5} \) 89. \(\frac{8}{15} \div \frac{4}{3} \)

134 Chapter 3 Solving Linear Equations
**Solving Equations by Using Multiplication and Division**

**What You’ll Learn**
- Solve equations by using multiplication.
- Solve equations by using division.

**How can equations be used to find how long it takes light to reach Earth?**

It may look like all seven stars in the Big Dipper are the same distance from Earth, but in fact, they are not. The diagram shows the distance between each star and Earth.

Light travels at a rate of about 5,870,000,000,000 miles per year. In general, the rate at which something travels times the time equals the distance \(rt = d\). The following equation can be used to find the time it takes light to reach Earth from the closest star in the Big Dipper.

\[rt = d\]

\[5,870,000,000,000t = 311,110,000,000,000\]

**SOLVE USING MULTIPLICATION** To solve equations such as the one above, you can use the **Multiplication Property of Equality**.

**Key Concept** **Multiplication Property of Equality**

- **Words**: If each side of an equation is multiplied by the same number, the resulting equation is true.
- **Symbols**: For any numbers \(a\), \(b\), and \(c\), if \(a = b\), then \(ac = bc\).
- **Examples**
  - \(6 = 6\)
  - \(9 = 9\)
  - \(10 = 10\)
  - \(6 \times 2 = 6 \times 2\)
  - \(9 \times (-3) = 9 \times (-3)\)
  - \(10 \times \frac{1}{2} = 10 \times \frac{1}{2}\)
  - \(12 = 12\)
  - \(-27 = -27\)
  - \(5 = 5\)

**Example 1** Solve Using Multiplication by a Positive Number

Solve \(\frac{t}{30} = \frac{7}{10}\). Then check your solution.

\[
\frac{t}{30} = \frac{7}{10} \quad \text{Original equation}
\]

\[
30(\frac{t}{30}) = 30(\frac{7}{10}) \quad \text{Multiply each side by 30.}
\]

\[
t = 21 \quad \text{or} \quad \frac{1}{30}(30t) = 7\text{ and } \frac{7}{10}(30) = 21
\]

(continued on the next page)

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**Workbook and Reproducible Masters**

- **Chapter 3 Resource Masters**
  - Study Guide and Intervention, pp. 149–150
  - Skills Practice, p. 151
  - Practice, p. 152
  - Reading to Learn Mathematics, p. 153
  - Enrichment, p. 154
  - Assessment, p. 205

- **Parent and Student Study Guide Workbook**, p. 21
- **Prerequisite Skills Workbook**, pp. 9–12, 51–52

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**Transparencies**

- 5-Minute Check Transparency 3-3
- Answer Key Transparencies

**Technology**

- Interactive Chalkboard
Building on Prior Knowledge

In Lesson 3-2, students learned that to solve an equation, the variable must be isolated on one side of the equation. Because the numbers were added to or subtracted from the variable, they added (or subtracted) the same quantity to each side of the equation to accomplish this. Ask students how the number is “connected” to the variable (multiplication). Then ask what is the opposite operation of multiplication.

SOLVE USING MULTIPLICATION

**In-Class Examples**

**Teaching Tip** Remind students that the goal is to multiply the coefficient of the variable by a number so that the resulting coefficient is 1.

1. Solve \( \frac{s}{12} = \frac{3}{4} \). Then check your solution. 9

2. Solve \(-3\frac{3}{8}k = 1\frac{4}{5} - \frac{8}{15}\)

3. Solve \(-75 = -15b\). 5

**Example 2** Solve Using Multiplication by a Fraction

Solve \( \left(2\frac{1}{4}\right)g = 1\frac{1}{2} \).

\[
\left(2\frac{1}{4}\right)g = 1\frac{1}{2} \\
\left(\frac{9}{4}\right)g = \frac{3}{2} \\
\frac{4}{9}\left(\frac{9}{4}\right)g = \frac{3}{2} \\
g = \frac{3}{2} \\
\]

You can write an equation to represent a real-world problem. Then use the equation to solve the problem.

**Example 3** Solve Using Multiplication by a Negative Number

Solve \(42 = -6m\).

\[
42 = -6m \\
-\frac{1}{6}(42) = -\frac{1}{6}(-6m) \\
-7 = m \\
\]

The solution is \(-7\).

**Example 4** Write and Solve an Equation Using Multiplication

**SPACE TRAVEL** Refer to the information about space travel at the left. The weight of anything on the moon is about one-sixth its weight on Earth. What was the weight of Neil Armstrong’s suit and life-support backpacks on Earth?

**Words** One sixth times the weight on Earth equals the weight on the moon.

**Variable** Let \(w\) = the weight on Earth.

**Equation** \(\frac{1}{6}w = 33\)

\[
\frac{1}{6}w = 33 \\
6\left(\frac{1}{6}w\right) = 6(33) \\
w = 198 \\
\]

The weight of Neil Armstrong’s suit and life-support backpacks on Earth was about 198 pounds.
**SOLVE USING DIVISION** The equation in Example 3, $42 = -6m$, was solved by multiplying each side by $-\frac{1}{6}$. The same result could have been obtained by dividing each side by $-6$. This method uses the **Division Property of Equality**.

### Key Concept
**Division Property of Equality**
- **Words**: If each side of an equation is divided by the same nonzero number, the resulting equation is true.
- **Symbols**: For any numbers $a$, $b$, and $c$, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
- **Examples**
  - $15 = 15$  $28 = 28$
  - $15 \cdot 15 = 28 \cdot 28$
  - $3 = 3$  $-7 = -7$
  - $5 = 5$  $-4 = -4$

### Example 5 Solve Using Division by a Positive Number
Solve $13s = 195$. Then check your solution.

1. $13s = 195$  **Original equation**
2. $13 \div 13$  **Divide each side by 13.**
3. $s = 15$  **Substitute 15 for $s$.**

**CHECK**

1. $13s = 195$  **Original equation**
2. $13(15) \div 195$  **Substitute 15 for $s$.**
3. $195 = 195$  **The solution is 15.**

### Example 6 Solve Using Division by a Negative Number
Solve $-3x = 12$.

1. $-3x = 12$  **Original equation**
2. $-3 \div -3$  **Divide each side by $-3$.**
3. $x = 4$  **Substitute 15 for $s$.**

The solution is $4$.

### Example 7 Write and Solve an Equation Using Division
Write an equation for the problem below. Then solve the equation.

**Negative eighteen times a number equals $-198$.**

- $-18n = -198$  **Original equation**
- $-18 \div -198$  **Divide each side by $-18$.**
- $n = 11$  **Check this result.**

The solution is 11.
### Concept Check

1. **Sample answer:**
   \[ 4x = -12 \]

2. **Dividing each side of an equation by a number is the same as multiplying each side of the equation by the number’s reciprocal.**

### GUIDED PRACTICE KEY

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<td>7</td>
</tr>
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<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

**Guided Practice**

Solve each equation. Then check your solution.

4. \[-2x = -84\]
5. \[\frac{5}{7} = -5\]
6. \[\frac{a}{3} = \frac{4}{9}\]
7. \[\frac{4k}{3} = \frac{8}{9} \Rightarrow \frac{k}{9} = \frac{1}{9}\]
8. \[3.15 = 1.5y \Rightarrow y = 2.1\]
9. \[\left(\frac{3}{4}\right)p = 2\frac{1}{2} \Rightarrow p = \frac{13}{10}\]

**Write an equation for each problem. Then solve the equation.**

10. Five times a number is 120. What is the number?
11. Two fifths of a number equals –24. Find the number.

**Application**

12. **GEOGRAPHY** The discharge of a river is defined as the width of the river times the average depth of the river times the speed of the river. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location? **10 m**

**Check for Understanding**

1. **OPEN ENDED** Write a multiplication equation that has a solution of \(-3\).

2. **Explain** why the Multiplication Property of Equality and the Division Property of Equality can be considered the same property.

3. **FIND THE ERROR** Casey and Juanita are solving \(8n = -72\).

   - **Casey**
     \[8n = -72\]
     \[8(6) = -72\]
     \[n = -9\]
   - **Juanita**
     \[8n = -72\]
     \[\frac{n}{8} = -9\]

Who is correct? Explain your reasoning.
33. Seven times a number equals –84. What is the number?  \(7n = -84; \quad n = -12\)
34. Negative nine times a number is –117. Find the number.  \(-9n = -117; \quad n = 13\)
35. One fifth of a number is 12. Find the number.  \(\frac{1}{5}n = 12; \quad n = 60\)
36. Negative three eighths times a number equals 12. What is the number?  \(-\frac{3}{8}n = 12; \quad n = -32\)
37. Two and one half times a number equals one and one fifth. Find the number.  \(2.5n = 1.2; \quad n = 0.48\)
38. One and one third times a number is –4.82. What is the number?  \(1.33\overline{3}n = -4.82; \quad n = -\frac{144}{43}\)

40. 50 people

42. WORLD RECORDS In 1993, a group of people in Utica, New York, made a very large round jelly doughnut which broke the world record for doughnut size. It weighed 1.5 tons and had a circumference of 50 feet. What was the diameter of the doughnut? (Hint: \(C = \pi d\)) about 16 ft

BASEBALL For Exercises 43–45, use the following information.
In baseball, if all other factors are the same, the speed of a four-seam fastball is faster than a two-seam fastball. The distance from the pitcher’s mound to home plate is 60.5 feet.
43. How long does it take a two-seam fastball to go from the pitcher’s mound to home plate? Round to the nearest hundredth. (Hint: \(rt = d\)) 0.48 s
44. How long does it take a four-seam fastball to go from the pitcher’s mound to home plate? Round to the nearest hundredth. 0.46 s
45. How much longer does it take for a two-seam fastball to reach home plate than a four-seam fastball? about 0.02 s

PHYSICAL SCIENCE For Exercises 46–49, use the following information.
In science lab, Devin and his classmates are asked to determine how many grams of hydrogen and how many grams of oxygen are in 477 grams of water. Devin used what he learned in class to determine that for every 8 grams of oxygen in water, there is 1 gram of hydrogen.
46. If \(x\) represents the number of grams of hydrogen, write an expression to represent the number of grams of oxygen. \(8x\)
47. Write an equation to represent the situation. \(x + 8x = 477\)
48. How many grams of hydrogen are in 477 grams of water? 53 g
49. How many grams of oxygen are in 477 grams of water? 424 g

50. CRITICAL THINKING If \(6y - 7 = 4\), what is the value of \(18y - 21?\) 12

www.algebra1.com/self_check_quiz

Enrichment, p. 154

Dissection Puzzles: Make the Square
In a dissection puzzle, you are to cut apart one figure using only straight cuts and then reassemble the pieces to make a new figure. The number of cuts will be equal to the number of sides of the new figure. However, for these puzzles, the cut lines are shown. This makes it easier for you to determine how the pieces were cut apart.

Cut apart each figure. Then rearrange the pieces to form a square.

1. 

4. 

Study Guide and Intervention, p. 149 (shown) and p. 150

Solve for Multiplication 1. What is the value of \(a\) if \(2a + 3 = 7?\) \(a = 2\)
2. What is the value of \(b\) if \(5b - 4 = 11?\) \(b = 3\)
3. What is the value of \(c\) if \(3c + 2 = 8?\) \(c = 2\)

In science lab, Devin and his classmates are asked to determine how many grams of oxygen are in 477 grams of water. How many grams of hydrogen are in 477 grams of water? 53 g

For Exercises 28 and 29, use the following information.

16. \(1.3 \times 7 = 9.1\) Find the number.
17. \(1.3 \times 12 = 15.6\) Find the number.
18. \(1.3 \times 17 = 22.1\) Find the number.
19. \(1.3 \times 22 = 28.6\) Find the number.
20. \(1.3 \times 27 = 33.9\) Find the number.

Write an equation for each problem. Then solve the equation.

22. What is the value of \(n\) if \(2n + 3 = 7?\) \(n = 2\)
23. What is the value of \(n\) if \(2n - 3 = 7?\) \(n = 5\)
24. What is the value of \(n\) if \(n + 3 = 7?\) \(n = 4\)
25. What is the value of \(n\) if \(n - 3 = 7?\) \(n = 10\)
26. What is the value of \(n\) if \(n + 4 = 7?\) \(n = 3\)
27. What is the value of \(n\) if \(n - 4 = 7?\) \(n = 11\)
28. What is the value of \(n\) if \(n + 5 = 7?\) \(n = 2\)
29. What is the value of \(n\) if \(n - 5 = 7?\) \(n = 12\)

Use the following information.

For Exercises 28 and 29, use the following information.

38. A pica equals 1/72 of an inch, or approximately 0.013889 inches. What is the number of points in a pica? 12
39. A point is 1/72 of an inch, or approximately 0.013889 inches. What is the number of picas in a point? 0.013889

For Exercises 28 and 29, use the following information.

16. What is the average distance that a roller coaster ride is? 5.5 ft
17. The average speed of a roller coaster ride is 60 mph. What is the average time it takes to reach Earth from the farthest star in the Big Dipper? 8.2 s
18. What is the average distance that a roller coaster ride is? 5.5 ft
19. What is the average speed of a roller coaster ride? 60 mph

Reading to Learn Mathematics, p. 153

ELL

Pre-Activity

Here are some equations to be solved. How long has it taken light to reach Earth?

Read the introduction to Lesson 3-3 at the top of page 135 in your textbook.

1. If 
2. If 
3. If 
4. If 

Helping You Remember

If you need to remember something about a number line or a number line, use these tips.

Sample answer: Multiply each side of the equation by the reciprocal of the number. Then you can isolate a on the left side.
Lessons 3-1 through 3-3

The key to solving multi-step equations in Lesson 3-4.

Equations is knowing the order of operations. Use the order of operations to find each value.

How can equations be used to find how long it takes light to reach Earth?

Include the following in your answer:

- an explanation of how to find the length of time it takes light to reach Earth from the closest star in the Big Dipper, and
- an equation describing the situation for the furthest star in the Big Dipper.

51. **Writing in Math** Answer the question that was posed at the beginning of the lesson. See margin.

How can equations be used to find how long it takes light to reach Earth?

Include the following in your answer:

- an explanation of how to find the length of time it takes light to reach Earth from the closest star in the Big Dipper, and
- an equation describing the situation for the furthest star in the Big Dipper.

52. The rectangle at the right is divided into 5 identical squares. If the perimeter of the rectangle is 48 inches, what is the area of each square?

(A) 4 in²  
(B) 9.8 in²  
(C) 16 in²  
(D) 23.04 in²

53. Which equation is equivalent to 4t = 20?

(A) −2t = −10  
(B) t = 80  
(C) 2t = 5  
(D) −8t = 40

54. m + 14 = 81  
55. d – 27 = −14  
56. 17 − (−w) = −55 − 72

57. Translate the following sentence into an equation. (Lesson 3-1)

Ten times a number a is equal to 5 times the sum of b and c. 10a = 5(b + c)

Find each product. (Lesson 2-3)

58. (−5)(12) −60  
59. (−2.93)(−0.03)  
60. (−4)(0)(−2)(−3) 0.00879

Graph each set of numbers on a number line. (Lesson 2-1) 61–64. See margin.

61. {−4, −3, −1, 3}  
62. {−8, −6, −4, −2, −1, 0, 1, 2, 4, 5, 6, 7, 8, 9, 10}

63. {integers less than −4}  
64. {integers less than 0 and greater than −6}

Name the property illustrated by each statement. (Lesson 1-6)

65. 67 + 3 = 3 + 67  
66. (5 · m) · n = 5 · (m · n)  
67. 15 − 9 = 26 + 12 − 3

68. 20  
69. 9  
70. 19

PREREQUISITE SKILL Students will learn how to solve multi-step equations in Lesson 3-4. The key to solving multi-step equations is knowing the order in which to “undo” operations. This order is the opposite of the usual order of operations. Use Exercises 67–70 to determine your students’ familiarity with order of operations.

**Answers**

51. You can use the distance formula and the speed of light to find the time it takes light from the stars to reach Earth. Answers should include the following.

- Solve the equation by dividing each side of the equation by 5,870,000,000,000. The answer is 53 years.
- The equation 5,870,000,000,000f = 821,800,000,000,000 describes the situation for the star in the Big Dipper farthest from Earth.
Solving Multi-Step Equations

You can use an equation model to solve multi-step equations.

Solve $3x + 5 = -7$.

1. **Step 1** Model the equation.
   
   Place 3 $x$ tiles and 5 positive 1 tiles on one side of the mat. Place 7 negative 1 tiles on the other side of the mat.

2. **Step 2** Isolate the $x$ term.
   
   Remove zero pairs.

3. **Step 3** Remove zero pairs.
   
   Group the tiles to form zero pairs and remove the zero pairs.

4. **Step 4** Group the tiles.
   
   Separate the tiles into 3 equal groups to match the 3 $x$ tiles. Each $x$ tile is paired with 4 negative 1 tiles. Thus, $x = -4$.

**Model** Use algebra tiles to solve each equation.

1. $2x - 3 = -9$
2. $3x + 5 = 14$
3. $3x - 2 = 10$
4. $-8 = 2x + 4$
5. $3 + 4x = 11$
6. $2x + 7 = 1$
7. $9 = 4x - 7$
8. $7 + 3x = -8$
9. **MAKE A CONJECTURE** What steps would you use to solve $7x - 12 = -61$? First add 12 to each side, and then divide each side by 7.

**Teaching Algebra with Manipulatives**

- pp. 11-12 (masters for algebra tiles)
- p. 16 (master for equation mat)
- p. 68 (student recording sheet)

**Glencoe Mathematics Classroom Manipulative Kit**

- algebra tiles
- equation mat

**Assess**

In Exercises 1–8, students should

- discover which side of the equation directs the method of solution by locating the variable, and
- understand that addition and subtraction is done before multiplication and division when isolating the variable.

In Exercise 9, have students discuss how the steps in solving the equation is similar to or different from the order of operations.
5-Minute Check Transparency 3-4  Use as a quiz or review of Lesson 3-3.

Mathematical Background notes are available for this lesson on p. 118C.

How can equations be used to estimate the age of an animal?

Ask students:

• What does the number 8 represent in the expression $8 + 12a$? Eight inches, the length of an alligator hatchling.

• What does the $12a$ represent in the expression $8 + 12a$? Twelve represents the number of inches the alligator grows per year, and $a$ represents the number of years of growth.

• What does this expression assume about the growth of an alligator over its lifetime? An alligator continues to grow at a constant rate during its entire life.

Since 10 feet 4 inches equals $10(12) + 4$ or 124 inches, the equation $8 + 12a = 124$ can be used to estimate the age of the alligator in the photograph. Notice that this equation involves more than one operation.

WORK BACKWARD  Work backward is one of many problem-solving strategies that you can use. Here are some other problem-solving strategies.

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<tr>
<th>Problem-Solving Strategies</th>
<th>Statement Undo the Statement</th>
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<td>check for hidden assumptions</td>
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<tr>
<td>use a graph</td>
<td>identify the subgoals</td>
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Example 1  Work Backward to Solve a Problem

Solve the following problem by working backward.

After cashing her paycheck, Tara paid her father the $20 she had borrowed. She then spent half of the remaining money on a concert ticket. She bought lunch for $4.35 and had $10.55 left. What was the amount of the paycheck?

Start at the end of the problem and undo each step.

<table>
<thead>
<tr>
<th>Statement</th>
<th>$\text{Undo the Statement}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>She had $10.55 left.</td>
<td>$10.55$</td>
</tr>
<tr>
<td>She bought lunch for $4.35.</td>
<td>$10.55 + 4.35 = 14.90$</td>
</tr>
<tr>
<td>She spent half of the money on a concert ticket.</td>
<td>$14.90 \times 2 = 29.80$</td>
</tr>
<tr>
<td>She paid her father $20.</td>
<td>$29.80 + 20.00 = 49.80$</td>
</tr>
</tbody>
</table>

The paycheck was for $49.80. Check this answer in the context of the problem.
**SOLVE MULTI-STEP EQUATIONS** To solve equations with more than one operation, often called multi-step equations, undo operations by working backward.

### Example 2 Solve Using Addition and Division

Solve $7m - 17 = 60$. Then check your solution.

- **Original equation**
  $$7m - 17 = 60$$
- **Add 17 to each side.**
  $$7m = 77$$
- **Divide each side by 7.**
  $$m = 11$$

**CHECK**

- **Original equation**
  $$7m - 17 = 60$$
- **Substitute 11 for m.**
  $$77 - 17 = 60$$
- **The solution is 11.**

You have seen a multi-step equation in which the first, or leading, coefficient is an integer. You can use the same steps if the leading coefficient is a fraction.

### Example 3 Solve Using Subtraction and Multiplication

Solve $\frac{t}{8} + 21 = 14$. Then check your solution.

- **Original equation**
  $$\frac{t}{8} + 21 = 14$$
- **Subtract 21 from each side.**
  $$\frac{t}{8} = -7$$
- **Multiply each side by 8.**
  $$t = -56$$

**CHECK**

- **Original equation**
  $$\frac{t}{8} + 21 = 14$$
- **Substitute -56 for t.**
  $$\frac{-56}{8} + 21 = 14$$
- **The solution is -56.**

### Example 4 Solve Using Multiplication and Addition

Solve $\frac{p - 15}{9} = -6$.

- **Original equation**
  $$\frac{p - 15}{9} = -6$$
- **Multiply each side by 9.**
  $$p - 15 = -54$$
- **Add 15 to each side.**
  $$p = -39$$

The solution is -39.

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**Solve Using Multiplication and Addition**

### Example 2 Solve Using Addition and Division

Solve $7m - 17 = 60$. Then check your solution.

- **Original equation**
  $$7m - 17 = 60$$
- **Add 17 to each side.**
  $$7m = 77$$
- **Divide each side by 7.**
  $$m = 11$$

**CHECK**

- **Original equation**
  $$7m - 17 = 60$$
- **Substitute 11 for m.**
  $$77 - 17 = 60$$
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### Example 4 Solve Using Multiplication and Addition

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- **Original equation**
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- **Multiply each side by 9.**
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- **Add 15 to each side.**
  $$p = -39$$

The solution is -39.

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**Teach**

### In-Class Example

Solve the following problem by working backward.

Danny took some rope with him on his camping trip. He used 32 feet of rope to tie his canoe to a log on the shore. He then gave $\frac{1}{3}$ of the remaining rope to some fellow campers who also needed to tie a canoe. The next night, he used half of the remaining rope to secure his tent during a thunderstorm. On the last day, he used 7 feet as a fish stringer to keep the fish that he caught. After the camping trip, he had 9 feet left. How much rope did he have at the beginning of the camping trip? **80 ft**

---

**Solve Multi-Step Equations**

### In-Class Examples

2 Solve $5q - 13 = 37$. Then check your solution. **10**

3 Solve $\frac{s}{12} - 9 = -11$. Then check your solution. **-24**

4 Solve $\frac{r + 8}{-3} = -2$. **-2**
Concept Check

**Multi-Step Equations** Ask students: why do you multiply each side by 9 in Example 4 before you add 15 to each side, which is different from what you did in Examples 2 and 3?

This equation is like \( \frac{a}{9} = -6 \), where \( a = p - 15 \). To solve \( \frac{a}{9} = -6 \), you multiply each side by 9 first.

**In-Class Examples**

5. Write an equation for the problem below. Then solve the equation.

Eight more than five times a number is negative 62.

\[ 5n + 8 = -62; \quad n = -14 \]

**Teaching Tip** Ask students to explain why an equation to find consecutive odd integers looks like an equation to find consecutive even integers. Odds (or evens) are both calculated by adding 2 to the previous odd (or even).

6. **NUMBER THEORY** Write an equation for the problem below. Then solve the equation and answer the problem.

Find three consecutive odd integers whose sum is 57.

\[ n + (n + 2) + (n + 4) = 57 \] or \[ 3n + 6 = 57 \]. The consecutive integers are 17, 19, and 21.

**Example 5** Write and Solve a Multi-Step Equation

Write an equation for the problem below. Then solve the equation.

**Two-thirds of a number minus six is \(-10\).**

\[ \frac{2}{3}n - 6 = -10 \]  
Original equation

\[ \frac{2}{3}n - 6 + 6 = -10 + 6 \]  
Add 6 to each side.

\[ \frac{2}{3}n = -4 \]  
Simplify.

\[ \frac{3}{2} \left( \frac{2}{3}n \right) = \frac{3}{2} (-4) \]  
Multiply each side by \( \frac{3}{2} \).

\[ n = -6 \]  
Simplify.

The solution is \(-6\).

**Consecutive integers** are integers in counting order, such as 7, 8, and 9. Beginning with an even integer and counting by two will result in **consecutive even integers**. For example, \(-4, -2, 0, \) and \(2 \) are consecutive even integers. Beginning with an odd integer and counting by two will result in **consecutive odd integers**. For example, \(-3, -1, 1, 3 \) and \(5 \) are consecutive odd integers. The study of numbers and the relationships between them is called **number theory**.

**Example 6** Solve a Consecutive Integer Problem

**NUMBER THEORY** Write an equation for the problem below. Then solve the equation and answer the problem.

Find three consecutive even integers whose sum is \(-42\).

Let \( n \) = the least even integer.

Then \( n + 2 = \) the next greater even integer, and \( n + 4 = \) the greatest of the three even integers.

**The sum of three consecutive even integers** is \(-42\).

\[ n + (n + 2) + (n + 4) = -42 \]  
Original equation

\[ 3n + 6 = -42 \]  
Simplify.

\[ 3n + 6 - 6 = -42 - 6 \]  
Subtract 6 from each side.

\[ 3n = -48 \]  
Simplify.

\[ \frac{3n}{3} = \frac{-48}{3} \]  
Divide each side by 3

\[ n = -16 \]  
Simplify.

\[ n + 2 = -16 + 2 = -14 \]  
\[ n + 4 = -16 + 4 = -12 \]

The consecutive even integers are \(-16, -14, \) and \(-12\).

**CHECK** \(-16, -14, \) and \(-12\) are consecutive even integers.

\[ -16 + (-14) + (-12) = -42 \]  
\( \checkmark \)

---

**Study Tip**

**Representing Consecutive Integers**

You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of \( n \) — odd or even — that differs between the two expressions.

**DAILY INTERVENTION**

**Logical** Some students will identify with the orderly way in which multistep equations are solved by undoing the steps in reverse of the order of operations. Suggest that students make a table with the steps to follow to solve multistep equations. For example:

For \( ax + b = c \), subtract \( b \) from each side, then divide each side by \( a \).

For \( ax - b = c \), add \( b \) to each side, then divide each side by \( a \).
Guided Practice

1. OPEN ENDED Give two examples of multi-step equations that have a solution of -2. Sample answers: $2x + 3 = -1$, $3x - 1 = -7$
2. List the steps used to solve $\frac{w + 3}{5} - 4 = 6$.
3. Write an expression for the odd integer before odd integer $n$. $n - 2$
4. Justify each step. $\frac{4 - 2d}{5} + 3 - 3 = 9 - 3$
   a. ?
   b. ? Simplify.
   c. ?
   d. ? Simplify.
   e. ?
   f. ? Simplify.
   g. ?
   h. ? Simplify.

Guided Practice

Solve each problem by working backward.

5. A number is multiplied by seven, and then the product is added to 13. The result is 55. What is the number? 6
6. LIFE SCIENCE A bacteria population triples in number each day. If there are 2,187,000 bacteria on the seventh day, how many bacteria were there on the first day? 3000 bacteria

Solve each equation. Then check your solution.

7. $4x - 2 = -6$ $-1$
8. $18 = 5p + 3$ 3
9. $\frac{3}{2}x - 8 = 11$ $12\frac{2}{3}$
10. $\frac{b + 4}{2} = -17$ 30
11. $0.2n + 3 = 8.6$ 28
12. $3.1y - 1.5 = 5.32$ 2.2

Write an equation and solve each problem. 13. $12 - 2n = -34$; 23
14. Find three consecutive integers whose sum is 42.
   $n + (n + 1) + (n + 2) = 42$; 13, 14, 15

Application 15. WORLD CULTURES The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet? 12 letters

* indicates increased difficulty

Practice and Apply

Solve each problem by working backward.

16. A number is divided by 4, and then the quotient is added to 17. The result is 25. Find the number. 32
17. Nine is subtracted from a number, and then the difference is multiplied by 5. The result is 75. What is the number? 24

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Lesson 3-4 Solving Multi-Step Equations

About the Exercises...

Organization by Objective

Work Backward: 16–21
Solve Multi-Step Equations: 22–54

Odd/Even Assignments
Exercises 16–47 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 59–64 require the use of a graphing calculator.

Assignment Guide

Advanced: 16–50 even, 54–83 (optional: 84–89)
Answer

56. By using the length at birth, the amount of growth each year, and the current length, you can write and solve an equation to find the age of the animal. Answers should include the following.
- To solve the equation, subtract 8 from each side and then divide each side by 12.
- The alligator is about $9 \frac{2}{3}$ or 10 years old.

Solve each problem by working backward.

18. **GAMES** In the Trivia Bowl, each finalist must answer four questions correctly. Each question is worth twice as much as the question before it. The fourth question is worth $6000. How much is the first question worth? $750

19. **ICE SCULPTING** Due to melting, an ice sculpture loses one-half its weight every hour. After 8 hours, it weighs $\frac{5}{16}$ of a pound. How much did it weigh in the beginning? 80 lb

20. **FIREFIGHTING** A firefighter spraying water on a fire stood on the middle rung of a ladder. The smoke lessened, so she moved up 3 rungs. It got too hot, so she backed down 5 rungs. Later, she went up 7 rungs and stayed until the fire was out. Then, she climbed the remaining 4 rungs and went into the building. How many rungs does the ladder have? 19 rungs

21. **MONEY** Hugo withdrew some money from his bank account. He spent one third of the money for gasoline. Then he spent half of what was left for a haircut. He bought lunch for $6.55. When he got home, he had $13.45 left. How much did he withdraw from the bank? $60

Deliveries must be flagged. The maximum distance Ms. Jones can drive in one day and still stay within her budget? 450.5 mi
49. GEOMETRY The measures of the three sides of a triangle are consecutive even integers. The perimeter of the triangle is 54 centimeters. What are the lengths of the sides of the triangle? 16 cm, 18 cm, 20 cm

50. MOUNTAIN CLIMBING A general rule for those climbing more than 7000 feet above sea level is to allow a total of \(\frac{a - 7000}{2000} + 2\) weeks of camping during the ascension. In this expression, \(a\) represents the altitude in feet. If a group of mountain climbers have allowed for 9 weeks of camping in their schedule, how high can they climb without worrying about altitude sickness? 21,000 ft

SHOE SIZE For Exercises 51 and 52, use the following information.

If \(r\) represents the length of a person’s foot in inches, the expression \(2r - 12\) can be used to estimate his or her shoe size.

51. What is the approximate length of the foot of a person who wears size 8? 10 in.

52. Measure your foot and use the expression to determine your shoe size. How does this number compare to the size of shoe you are wearing? See students’ work.

53. SALES Trever Goetz is a salesperson who is paid a monthly salary of $500 plus a 2% commission on sales. How much must Mr. Goetz sell to earn $2000 this month? $75,000

54. GEOMETRY A rectangle is cut from the corner of a 10-inch by 10-inch of paper. The area of the remaining piece of paper is \(\frac{3}{4}\) of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle? 5 in.

55. CRITICAL THINKING Determine whether the following statement is sometimes, always, or never true.

The sum of two consecutive even numbers equals the sum of two consecutive odd numbers. Never

56. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How can equations be used to estimate the age of an animal?

Include the following in your answer:
• an explanation of how to solve the equation representing the age of the alligator, and
• an estimate of the age of the alligator.

57. Which equation represents the following problem? B

Fifteen minus three times a number equals negative twenty-two. Find the number.

A 3n - 15 = -22
B 15 - 3n = -22
C 3(n - 15) = -22
D 3(n + 15) = -22

58. Which equation has a solution of -5? D

A 2n - 6 = 4
B 3n - 7 = 2
C 4n + 8 = 4
D 3a + 7 = 8
E 2a + 19 = 16

Lesson 3-4 Solving Multi-Step Equations

Consecutive Integer Problems

Many types of problems and solutions involve the idea of consecutive integers. Reading how to represent these integers algebraically can help solve the problem.

Strategy

Strategy Find four consecutive odd integers whose sum is 94.

The odd integer can be written as \(2n + 1\), where \(n\) is any integer.

If \(2n + 1\) is the first odd integer, then \(2n + 3, 2n + 5, 2n + 7\) are the next largest odd integers, and so on.

Write an equation to solve the problem.

\[4(2n + 1) = 94\]

Solve the equation.

\[8n + 4 = 94\]

\[8n = 90\]

\[n = 11.25\]

The solution is \(n = 11\), \(22\), \(23\), and \(24\).

Write an equation for each problem. Then solve.

1. Complete the solution to the problem in the example.
Open-Ended Assessment
Speaking Have one student volunteer read a word problem similar to Example 1, while another volunteer describes how to solve the problem, either by working backward or by writing and solving a two-step equation.

Getting Ready for Lesson 3-5
PREREQUISITE SKILL Students will learn how to solve equations with the variable on both sides in Lesson 3-5. This requires students to add or subtract expressions involving a variable from each side of the equation. Use Exercises 84–89 to determine your students’ familiarity with simplifying expressions. Students need to know how to simplify these sums or differences readily.

Maintain Your Skills

Mixed Review
Solve each equation. Then check your solution. (Lesson 3-3)
65. \(-7t = 91\) 66. \(\frac{t}{15} = -8\) 67. \(-\frac{2}{3}b = -1\frac{1}{2}\)

TRANSPORTATION For Exercises 68 and 69, use the following information.
In the year 2000, there were 18 more models of sport utility vehicles than there were in the year 1990. There were 47 models of sport utility vehicles in 2000. (Lesson 3-2)
68. Write an addition equation to represent the situation. \(m + 18 = 47\)
69. How many models of sport utility vehicles were there in 1990? 29 models

Find the odds of each outcome if you spin the spinner at the right. (Lesson 2-6)
70. spinning a number divisible by 3 \(1:3\)
71. spinning a number equal to or greater than 5 \(1:1\)
72. spinning a number less than 7 \(3:1\)

Find each quotient. (Lesson 2-4)
73. \(-\frac{6}{7} ÷ 3 = -\frac{2}{7}\) 74. \(\frac{2}{8} ÷ \frac{1}{12}\) 75. \(-\frac{3a + 16}{4}\)

Use the Distributive Property to find each product. (Lesson 1-5)
77. \(17 \cdot 9\) 78. \(13(101)\) 79. \(16(\frac{1}{4})\) 80. \(18(\frac{2}{9})\)

Write an algebraic expression for each verbal expression. (Lesson 1-1)
81. the product of 5 and \(m\) plus half of \(a\) \(5m + \frac{1}{2}a\)
82. the sum of 3 and \(b\) divided by \(y\) \((3 + b) ÷ y\)
83. the sum of 3 times \(a\) and the square of \(b\) \(3a + b^2\)

Getting Ready for the Next Lesson
PREREQUISITE SKILL Simplify each expression. (To review simplifying expressions, see Lesson 1-5.)
84. \(5d - 2d = 3d\) 85. \(11m - 5m = 6m\)
86. \(8t + 6t = 14t\) 87. \(7g - 15g = -8g\)
88. \(-9f + 6f = -3f\) 89. \(-3m + (-7m) = -10m\)
Solving Equations with the Variable on Each Side

What You’ll Learn

• Solve equations with the variable on each side.
• Solve equations involving grouping symbols.

Vocabulary
• identity

How can an equation be used to determine when two populations are equal?

In 1995, there were 18 million Internet users in North America. Of this total, 12 million were male, and 6 million were female. During the next five years, the number of male Internet users on average increased 7.6 million per year, and the number of female Internet users increased 8 million per year. If this trend continues, the following expressions represent the number of male and female Internet users x years after 1995.

Male Internet Users: \(12 + 7.6x\)

Female Internet Users: \(6 + 8x\)

The equation \(12 + 7.6x = 6 + 8x\) represents the time at which the number of male and female Internet users are equal. Notice that this equation has the variable \(x\) on each side.

VARIABLES ON EACH SIDE

Many equations contain variables on each side. To solve these types of equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

Example 1 Solve an Equation with Variables on Each Side

Solve \(-2 + 10p = 8p - 1\). Then check your solution.

\[
\begin{align*}
-2 + 10p &= 8p - 1 & \text{Original equation} \\
-2 + 10p - 8p &= 8p - 1 - 8p & \text{Subtract} 8p \text{ from each side.} \\
-2 + 2p &= -1 & \text{Simplify.} \\
-2 + 2p + 2 &= -1 + 2 & \text{Add} 2 \text{ to each side.} \\
2p &= 1 & \text{Simplify.} \\
\frac{2p}{2} &= \frac{1}{2} & \text{Divide each side by} 2. \\
p &= \frac{1}{2} \text{ or } 0.5 & \text{Simplify.}
\end{align*}
\]

CHECK

\[
\begin{align*}
-2 + 10p &= 8p - 1 & \text{Original equation} \\
-2 + 10(0.5) &= 8(0.5) - 1 & \text{Substitute} 0.5 \text{ for } p. \\
-2 + 5 &= 4 - 1 & \text{Multiply.} \\
3 &= 3 & \text{The solution is} \frac{1}{2} \text{ or } 0.5.
\end{align*}
\]
GROUPING SYMBOLS  When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 2  Solve an Equation with Grouping Symbols

Solve \(4(2r - 8) = \frac{1}{7}(49r + 70)\). Then check your solution.

1. \(4(2r - 8) = \frac{1}{7}(49r + 70)\)  
   Original equation
2. \(8r - 32 = 7r + 10\)  
   Distributive Property
3. \(8r - 32 - 7r = 7r + 10 - 7r\)  
   Subtract 7r from each side.
4. \(r - 32 = 10\)  
   Simplify.
5. \(r - 32 + 32 = 10 + 32\)  
   Add 32 to each side.
6. \(r = 42\)  
   Simplify.

CHECK

1. \(4(2r - 8) = \frac{1}{7}(49r + 70)\)  
   Original equation
2. \(4(2(42) - 8) = \frac{1}{7}(49(42) + 70)\)  
   Substitute 42 for \(r\).
3. \(4(84 - 8) = \frac{1}{7}(2058 + 70)\)  
   Multiply.
4. \(4(76) = \frac{1}{7}(2128)\)  
   Add and subtract.
5. \(304 = 304\)  

The solution is 42.

Some equations with the variable on each side may have no solution. That is, there is no value of the variable that will result in a true equation.

Example 3  No Solutions

Solve \(2m + 5 = 5(m - 7) - 3m\).

1. \(2m + 5 = 5(m - 7) - 3m\)  
   Original equation
2. \(2m + 5 = 5m - 35 - 3m\)  
   Distributive Property
3. \(2m + 5 = 2m - 35\)  
   Simplify.
4. \(2m + 5 - 2m = 2m - 35 - 2m\)  
   Subtract 2m from each side.
5. \(5 = -35\)  
   This statement is false.

Since \(5 = -35\) is a false statement, this equation has no solution.

Example 4  An Identity

Solve \(3(r + 1) - 5 = 3r - 2\).

1. \(3(r + 1) - 5 = 3r - 2\)  
   Original equation
2. \(3r + 3 - 5 = 3r - 2\)  
   Distributive Property
3. \(3r - 2 = 3r - 2\)  
   Reflexive Property of Equality

Since the expressions on each side of the equation are the same, this equation is an identity. The statement \(3(r + 1) - 5 = 3r - 2\) is true for all values of \(r\).

D A I L Y  I N T E R V E N T I O N

Differentiated Instruction

Kinesthetic  Students will benefit from manipulating or moving objects to help them solve equations with variables on both sides. Allow students to use equation mats and algebra tiles to model simple equations. Manipulating the tiles will give students a different way to learn the concepts in this lesson.
Standardized Test Practice

Example 5 Use Substitution to Solve an Equation

Multiple-Choice Test Item

Solve $2(b - 3) + 5 = 3(b - 1)$.

A. $-2$

B. $2$

C. $-3$

D. $3$

Read the Test Item
You are asked to solve an equation.

Solve the Test Item
You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

A. $2(b - 3) + 5 = 3(b - 1)$

B. $2(b - 3) + 5 = 3(b - 1)$

$2(-2 - 3) + 5 \neq 3(-2 - 1)$

$2(-5) + 5 \neq 3(-3)$

$-10 + 5 \neq -9$

$-5 \neq -9$

Since the value 2 results in a true statement, you do not need to check $-3$ and $3$. The answer is B.

Check for Understanding

Concept Check

1. Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

   a. $2(g + 5) = 22$

   $2g + 5 = 22$

   $2g + 5 - 5 = 22 - 5$

   $2g = 17$

   $g = 8.5$

   b. $5d = 2d - 18$

   $3d = -18$

   Incorrect; to eliminate $-6z$ on the right side of the equals sign, $6z$ must be added to each side of the equation; 1.

   c. $-6z + 13 = 7z$

   $-6z + 13 - 6z = 7z - 6z$

   $13 = z$

   $d = -6$

   $\frac{3d}{3} = -\frac{18}{3}$

   correct

   www.algebra1.com/extra_examples

Example 5 Caution students to be very careful when checking possible answers on test items. It is very easy to make a substitution or arithmetic error when checking possible answers. Often, some of the incorrect possible answers are similar to the correct answers, such as $-2$ and $2$ in Example 5. It would be easy to substitute $2$ in the equation and then inadvertently choose $-2$ as the answer.
2. If both sides of the equation are always equal, the equation is an identity.

3. OPEN ENDING Find a counterexample to the statement all equations have a solution. Sample answer: $2x - 5 = 2x + 5$

4. Justify each step.
   \[ 6n + 7 = 8n - 13 \]
   a. Subtract $6n$ from each side.
   \[ 7 = 2n - 13 \]
   b. Simplify.
   \[ 7 + 13 = 2n + 13 + 13 \]
   c. ?
   \[ 20 = 2n \]
   d. Simplify.
   \[ \frac{20}{2} = \frac{2n}{2} \]
   e. ?
   \[ 10 = n \]
   f. Simplify.

Solve each equation. Then check your solution.
5. $20c + 5 = 5c + 65$
6. $\frac{3}{4} - \frac{1}{4} = \frac{1}{2} - \frac{3}{4}$
7. $3(a - 5) = -6$
8. $7 - 3r = r - 4(2 + r)$
9. $6 = 3 + 5(d - 2)$
10. $\frac{c + 1}{8} = \frac{c}{4}$
11. $5h - 7 = 5(h - 2) + 3$
12. $5.4w + 8.2 = 9.8w - 2.8$

**Standardized Test Practice**
13. Solve $75 - 9t = 5(-4 + 2t)$. D
   - A. $-5$
   - B. $-4$
   - C. $4$
   - D. $5$

**About the Exercises...**

**Organization by Objective**
- Variables on Each Side: 14–21, 30, 31, 34–37, 40, 41, 46
- Grouping Symbols: 22–29, 32, 33, 38, 39, 42–45, 47, 48

**Odd/Even Assignments**
Exercises 14–47 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**
- Basic: 15–45 odd, 49–75
- Average: 15–47 odd, 49–75
- Advanced: 14–46 even, 48–67 (optional: 68–75)
30. \( \frac{3}{2}y - y = 4 + \frac{1}{2}y \) no solution
31. \( 3 + \frac{2}{3}t = 11 - \frac{2}{3}t \)
32. \( \frac{1}{2}(7 + 3q) = 8 - 2 \)
33. \( \frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4) - 4 \)
34. \( 28 - 2.2x = 11.6x + 262.6 - 17 \)
35. \( 1.03y - 4 = -2.15y + 8.72 \)
36. \( 18 - 3.8t = 7.36 - 1.9t \)
37. \( 13.7v - 6.5 = -2.3v + 8.3 \)
38. \( 2[s + 3(s - 1)] = 18 \)

40. One half of a number increased by 16 is four less than two thirds of the number. Find the number. 120
41. The sum of one half of a number and 6 equals one third of the number. What is the number? -36

42. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers. 17, 19
43. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers? 26, 28, 30
44. **HEALTH** When exercising, a person’s pulse rate should not exceed a certain limit, which depends on his or her age. This maximum rate is represented by the expression 0.8(220 - a), where a is age in years. Find the age of a person whose maximum pulse is 150. 30 years

45. **HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. What is the name of a nail that is \( \frac{21}{2} \) inches long? 8-penny

46. **TECHNOLOGY** About 4.9 million households had one brand of personal computers in 2001. The use of these computers grew at an average rate of 0.275 million households a year. In 2001, about 2.5 million households used another type of computer. The use of these computers grew at an average rate of 0.7 million households a year. How long will it take for the two types of computers to be in the same number of households? About 5.6 yr

47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.

48. **ENERGY** Use the information on energy at the left. The amount of energy \( E \) in BTUs needed to raise the temperature of water is represented by the equation \( E = w(t_f - t_o) \). In this equation, \( w \) represents the weight of the water in pounds, \( t_f \) represents the final temperature in degrees Fahrenheit, and \( t_o \) represents the original temperature in degrees Fahrenheit. A 50-gallon water heater is 60% efficient. If 10 cubic feet of natural gas are used to raise the temperature of water with the original temperature of 50°F, what is the final temperature of the water? (One gallon of water weighs about 8 pounds.) 68°F

49. **Sample answer.** \( 3(x + 1) = x - 1 \)

50. **CRITICAL THINKING** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of -2.

www.algebra1.com/self_check_quiz

**Enrichment, p. 166**

**Identities**

An equation that is true for every value of the variable is called an identity. When you try to solve an identity, you end up with a statement that is always true. Here is an example.

**Example**

Solve \( 8 - 5 = 3(3 + 3) \).

- \( 8 - 5 = 3(3 + 3) \)
- \( 3 - 5 = 3(3 + 3) \)
- \( 3 - 5 = 3(6) \)
- \( 3 - 5 = 18 \)

**Solution**

State whether each equation is an identity. If it is not, find its solution.

**Questions**

- **a.** \( x + 2 = 2x + 2 \) the solution is -2.
- **b.** \( 2x + 4 = 2x + 4 \) all numbers are solutions.
- **c.** \( x = 2 \) there are no solutions.

- **d.** In question 3, one of the equations from the last section was an identity. Which equation was it?

- **Helping You Remember**

An equation with variable is an identity when the equation is always true. In other words, the expressions on the left and right sides always have the same value. Look up the last identity in the dictionary. Write all the definitions that are similar to the mathematical definition. See students' work.

**Study Guide and Intervention, p. 161 (shown) and p. 162**

**Variables on Each Side**

To solve equations with variables on each side, first use the Addition or the Subtraction Property of Equality to get all variables on one side of the equation. Then solve the equation. For example, consider the equation

\[ 5x - 4 = 3x + 8 \]

Subtract 3x from each side to get all variables on one side of the equation.

\[ 2x - 4 = 8 \]

Add 4 to both sides to get all constants on the other side.

\[ 2x = 12 \]

Divide each side by 2 to get the variable by itself.

\[ x = 6 \]

The solution is 6.

**Example**

Solve \( 5x - 4 = 3x + 8 \).

- \( 5x - 4 = 3x + 8 \)
- \( 5x - 3x = 8 + 4 \)
- \( 2x = 12 \)
- \( x = 6 \)

The solution is 6.

**Skills Practice, p. 163 and Practice, p. 164 (shown)**

Solve each equation. Then check your solution.

- \( 2x + 4 = 6x - 2 \)
- \( 6x - 4 = 2x - 2 \)
- \( 3(x - 2) = x + 1 \)
- \( 3x - 2 = 4 - x + 2 \)

**Facts and Figures**

- \( 19 \) is a prime number.
- It is the sum of two consecutive prime numbers.
- It is the difference of two consecutive even numbers.
- It is the product of two consecutive odd numbers.
- It is the sum of the first four consecutive odd numbers.
- It is the sum of the first six consecutive odd numbers.

- **ELL**

You can be asked to determine whether two populations are equal. Read the introduction to Lesson 3.5 at the top of page 158 in your textbook.

**Pre-Activity**

To find the equation 12 = 7b + 4, what are the steps that should be followed?

- **ELL**

You can be asked to determine whether two populations are equal. Read the introduction to Lesson 3.5 at the top of page 158 in your textbook.

**Reading Lesson**

- **ELL**

You can be asked to determine whether two populations are equal. Read the introduction to Lesson 3.5 at the top of page 158 in your textbook.

**Resources**

**Skills Practice, p. 163 and Practice, p. 164 (shown)**

Solve each equation. Then check your solution.

- \( 2x + 4 = 6x - 2 \)
- \( 6x - 4 = 2x - 2 \)
- \( 3(x - 2) = x + 1 \)
- \( 3x - 2 = 4 - x + 2 \)

Lesson 3-5 Solving Equations with the Variable on Each Side 153

Lesson 3-5 Solving Equations with the Variable on Each Side 153
4 Assess

Open-Ended Assessment

Writing Have students solve the equation $3x + 2 = 5x - 8$. Beside each step, have students write one or two sentences explaining and justifying their method.

Getting Ready for Lesson 3-6

PREREQUISITE SKILL Students will learn about ratios and proportions in Lesson 3-6. Ratios are fractions and proportions are equations involving fractions. Students should be able to simplify fractions readily before beginning Lesson 3-6. Use Exercises 68–75 to determine your students’ familiarity with simplifying fractions.

Assessment Options

Quiz (Lessons 3-4 and 3-5) is available on p. 205 of the Chapter 3 Resource Masters.

Mid-Chapter Test (Lessons 3-1 through 3-5) is available on p. 207 of the Chapter 3 Resource Masters.

Answers

50. Set two expressions equal to each other and solve the equation. Answers should include the following.

- The steps used to solve the equation are (1) subtract 7.6x from each side, (2) subtract 6 from each side, and (3) divide each side by 0.4.
- The number of male and female Internet users will be the same in 2010.
- If two expressions that represent the growth in use of two items are set equal to each other, the solution to the equation can predict when the number of items in use will be equal.

51. Solve $8x - 3 = 5(2x + 1)$. D

52. Solve $5n + 4 = 7(n + 1) - 2n$. C

Maintain Your Skills

Mixed Review

Solve each equation. Then check your solution. (Lesson 3-4)

53. $\frac{x}{2} - 6 = 14$ 90

54. $\frac{x - 3}{7} = -2$ 11

55. $5 - 9w = 23$ 2

HEALTH For Exercises 56 and 57, use the following information. (Lesson 3-3)

56. Write a multiplication equation representing the number of Calories C burned if Ebony pushes the lawn mower for m minutes. $C = 4.5m$

57. How long will it take Ebony to burn 150 Calories mowing the lawn? $33\frac{1}{3}$ min

Use each set of data to make a line plot. (Lesson 2-5)

58. 13, 15, 11, 15, 16, 17, 12, 12, 13, 15, 16, 15

59. 22, 25, 19, 21, 22, 24, 22, 25, 28, 21, 24, 22

Find each sum or difference. (Lesson 2-2)

60. $-10 + (-17)$ 27

61. $-12 - (-8)$ 4

62. $6 - 14$ 5

Write a counterexample for each statement. (Lesson 1-7)

63. If the sum of two numbers is even, then both addends are even.

64. If you are baking cookies, you will need chocolate chips. Sample answer: You could bake sugar cookies, which do not require chocolate chips.

Evaluate each expression when $a = 5$, $b = 8$, $c = 7$, $x = 2$, and $y = 1$. (Lesson 1-2)

65. $\frac{3x^2}{b + c}$ 5

66. $x(a + 2b) - y$ 41

67. $5(x + 2y) - 4ax$ 0

58. 10 12 14 16 18 20

59. 18 20 22 24 26 28
5-Minute Check Transparency 3-6 Use as a quiz or review of Lesson 3-5.

Mathematical Background notes are available for this lesson on p. 118D.

How are ratios used in recipes?
Ask students:

- The recipe is for 4 servings. If you want to double the recipe to make 8 servings, what would you do to the amounts for each ingredient? Double each amount.
- What would you do to the amount for each ingredient if you wanted to make enough frozen yogurt for two servings? Halve the amount of each ingredient.
- Suppose you want to make enough frozen yogurt for six servings. How much more is six servings than four? Six is 1 1/2 times as much as four.
- By what number would you multiply the amount of each ingredient to make six servings? You would multiply each amount by 1 1/2.

Study Tip

Reading Math
A ratio that is equivalent to a whole number is written with a denominator of 1.

### Ratios and Proportions

A ratio is a comparison of two numbers by division. The ratio of \( x \) to \( y \) can be expressed in the following ways.

\[
\frac{x}{y} \quad x : y \quad \frac{x}{y}
\]

Ratios are often expressed in simplest form. For example, the recipe above states that for 4 servings, you need 2 cups of milk. The ratio of servings to milk may be written as 4 to 2, 4:2, or \( \frac{4}{2} \). Written in simplest form, the ratio of servings to milk can be written as 2 to 1, 2:1, or \( \frac{2}{1} \).

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is \( \frac{8}{4} \). When this ratio is simplified, the ratio is \( \frac{2}{1} \) Notice that this ratio is equal to the original ratio.

An equation stating that two ratios are equal is called a proportion. So, we can state that \( \frac{2}{1} = \frac{8}{4} \) is a proportion.

#### Example 1 Determine Whether Ratios Form a Proportion

Determine whether the ratios \( \frac{4}{5} \) and \( \frac{24}{30} \) form a proportion.

\[
\begin{align*}
\frac{4}{5} \div 1 &= \frac{4}{5} \\
\frac{24}{30} \div 6 &= \frac{4}{5}
\end{align*}
\]

The ratios are equal. Therefore, they form a proportion.
2 Teach

RATIOS AND PROPORTIONS

In-Class Examples

Teaching Tip Students may notice that two ratios form a proportion when the numerator and denominator of the less simplified ratio are products of the numerator and denominator of the more simplified ratio by the same factor. In Example 1,
\[
\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}.
\]

1 Determine whether the ratios \(\frac{7}{8}\) and \(\frac{49}{56}\) form a proportion.

The ratios are equal when expressed in simplest form. Therefore, they form a proportion.

2 Use cross products to determine whether each pair of ratios form a proportion.

a. \(\frac{0.25}{0.6}, \frac{1.25}{2}\): not a proportion

b. \(\frac{4}{5}, \frac{16}{20}\): a proportion

SOLVE PROPORTIONS

In-Class Example

156 Chapter 3 Solving Linear Equations

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

Example 2 Use Cross Products

Use cross products to determine whether each pair of ratios form a proportion.

a. \(\frac{0.4}{0.8}, \frac{0.7}{1.4}\)

\[
\frac{0.4}{0.8} = \frac{0.7}{1.4} \quad \text{Write the equation.}
\]

\[
0.4(1.4) = 0.8(0.7) \quad \text{Find the cross products.}
\]

\[
0.56 = 0.56 \quad \text{Simplify.}
\]

The cross products are equal, so \(\frac{0.4}{0.8} = \frac{0.7}{1.4}\). Since the ratios are equal, they form a proportion.

b. \(\frac{6}{8}, \frac{24}{28}\)

\[
\frac{6}{8} = \frac{24}{28} \quad \text{Write the equation.}
\]

\[
6(28) = 8(24) \quad \text{Find the cross products.}
\]

\[
168 \neq 192 \quad \text{Simplify.}
\]

The cross products are not equal, so \(\frac{6}{8} \neq \frac{24}{28}\). The ratios do not form a proportion.

In the proportion \(\frac{0.4}{0.8} = \frac{0.7}{1.4}\) above, 0.4 and 1.4 are called the extremes, and 0.8 and 0.7 are called the means.

Key Concept

Means-Extremes Property of Proportion

- Words In a proportion, the product of the extremes is equal to the product of the means.

- Symbols \(\frac{a}{b} = \frac{c}{d}\), then \(ad = bc\).

- Examples Since \(\frac{2}{4} = \frac{1}{2}\), \(2(2) = 4(1)\) or \(4 = 4\).

SOLVE PROPORTIONS

You can write proportions that involve a variable. To solve the proportion, use cross products and the techniques used to solve other equations.

Example 3 Solve a Proportion

Solve the proportion \(\frac{n}{12} = \frac{3}{8}\).

\[
\frac{n}{12} = \frac{24}{16} \quad \text{Original equation}
\]

\[
16n = 15(24) \quad \text{Find the cross products.}
\]

\[
16n = 360 \quad \text{Simplify.}
\]

\[
\frac{16n}{16} = \frac{360}{16} \quad \text{Divide each side by 16.}
\]

\[
n = 22.5 \quad \text{Simplify.}
\]

Tips for New Teachers

ELL Explain to students that the definitions of extremes and means are not arbitrary. In the proportion \(\frac{a}{b} = \frac{c}{d}\), \(a\) and \(d\) are the extremes, and \(b\) and \(c\) are the means. Remind students that ratios can also be written in the form \(xy\). If you rewrite the proportion above in this form, you have \(a:b = c:d\). Looking at this proportion, \(a\) and \(d\) are the extremes because they are on the outside, and extreme is a synonym for outside. Similarly, \(b\) and \(c\) are the means because they are in the middle, and mean is often a synonym for middle.
The ratio of two measurements having different units of measure is called a **rate**. For example, a price of $1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of $30,000 per year are all rates. Proportions are often used to solve problems involving rates.

**Example 4 Use Rates**

**BICYCLING** Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

**Explore** Let \( m \) represent the number of miles Trent can ride in 6 hours.

**Plan** Write a proportion for the problem.

\[
\frac{m}{6} = \frac{30}{4}
\]

**Solve**

\[
\begin{align*}
30(6) &= 4(m) \\
180 &= 4m \\
45 &= m
\end{align*}
\]

**Examine** If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6 \( \times \) 7.5 or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

A ratio or rate called a **scale** is used when making a model or drawing of something that is too large or too small to be conveniently drawn at actual size. The scale compares the model to the actual size of the object using a proportion. Maps and blueprints are two commonly used scale drawings.

**Example 5 Use a Scale Drawing**

**CRATER LAKE** The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about \( \frac{3}{4} \) inches. What is the distance between these two places?

Let \( d \) represent the actual distance.

\[
\begin{align*}
\text{scale} &\rightarrow \frac{2}{9} = \frac{\frac{3}{4}}{d} \\
\text{actual} &\rightarrow \frac{1}{9} = \frac{\frac{3}{4}}{d}
\end{align*}
\]

Find the cross products.

\[
2(d) = 9\left(\frac{3}{4}\right)
\]

Simplify.

\[
2d = \frac{27}{4}
\]

Divide each side by 2.

\[
d = \frac{27}{8} \text{ or } 3\frac{3}{8}
\]

The actual distance is about \( 3\frac{3}{8} \) miles.

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**In-Class Examples**

**4 BICYCLING** The gear on a bicycle is 8:5. This means that for every 8 turns of the pedals, the wheel turns 5 times. Suppose the bicycle wheel turns about 2435 times during a trip. How many times would you have to crank the pedals during the trip? About 3896 times

**5 MAP** In a road atlas, the scale for the map of Connecticut is 5 inches = 41 miles. The scale for the map of Texas is 5 inches = 144 miles. What are the distances in miles represented by 2 \( \frac{1}{2} \) inches on each map? Connecticut 20 \( \frac{1}{2} \) mi; Texas 72 mi

---

**About the Exercises**

**Organization by Objective**

- **Ratios and Proportions:** 11–18
- **Solve Proportions:** 19–35

**Odd/Even Assignments**

Exercises 11–34 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

- **Basic:** 11–23 odd, 31, 33, 36–58
- **Average:** 11–35 odd, 36–58
- **Advanced:** 12–34 even, 36–54 (optional: 55–58)

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**In-Class Examples**

**4 BICYCLING** The gear on a bicycle is 8:5. This means that for every 8 turns of the pedals, the wheel turns 5 times. Suppose the bicycle wheel turns about 2435 times during a trip. How many times would you have to crank the pedals during the trip? About 3896 times

**5 MAP** In a road atlas, the scale for the map of Connecticut is 5 inches = 41 miles. The scale for the map of Texas is 5 inches = 144 miles. What are the distances in miles represented by 2 \( \frac{1}{2} \) inches on each map? Connecticut 20 \( \frac{1}{2} \) mi; Texas 72 mi
Concept Check

1. OPEN ENDED Find an example of ratios used in advertisements.
2. Explain the difference between a ratio and a proportion.
3. Describe how to solve a proportion if one of the ratios contains a variable.

Guided Practice

Use cross products to determine whether each pair of ratios forms a proportion.

Write yes or no.

4. \( \frac{4}{12} \) yes
5. \( \frac{16}{8} \) yes
6. \( \frac{21}{5} \) no

Solve each proportion. If necessary, round to the nearest hundredth.

7. \( \frac{3}{6} = \frac{6}{12} \) yes
8. \( \frac{12}{a} = \frac{15}{a} \) yes
9. \( \frac{0.6}{1.1} = \frac{n}{8.47} \) yes

SPORTS For Exercises 17 and 18, use the graph at the right.

17. Write a ratio of the number of gold medals won to the total number of medals won for each country.
18. Do any two of the ratios you wrote for Exercise 17 form a proportion? If so, explain the real-world meaning of the proportion. No; all of these ratios formed a proportion, the two countries would have the same part of their medals as gold medals.

Check for Understanding

1. See students’ work.
2–3. See margin.

Practice and Apply

Use cross products to determine whether each pair of ratios forms a proportion.

Write yes or no.

11. \( \frac{3}{2} \), \( \frac{15}{5} \) yes
12. \( \frac{7}{10} \), \( \frac{14}{20} \) yes
13. \( \frac{2}{3} \), \( \frac{3}{4} \) no

Solve each proportion.

14. \( \frac{2.88}{1.6} = \frac{6}{x} \) yes
15. \( \frac{1.23}{15} = \frac{21}{180} \) yes

Extra Practice

See page 287.

USA TODAY Snapshots®

USA stands atop all-time medals table

The USA, which led the 2000 Summer Olympics with 97 medals, has dominated the medal standings over the years. The all-time Summer Olympics medal standings:

<table>
<thead>
<tr>
<th>Country</th>
<th>Gold</th>
<th>Silver</th>
<th>Bronze</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>216</td>
<td>525</td>
<td>446</td>
</tr>
<tr>
<td>USSR/Russia</td>
<td>498</td>
<td>316</td>
<td>229</td>
</tr>
<tr>
<td>Germany</td>
<td>374</td>
<td>352</td>
<td>248</td>
</tr>
<tr>
<td>GB</td>
<td>180</td>
<td>628</td>
<td>596</td>
</tr>
<tr>
<td>Italy</td>
<td>179</td>
<td>479</td>
<td>231</td>
</tr>
<tr>
<td>Sweden</td>
<td>136</td>
<td>469</td>
<td>201</td>
</tr>
</tbody>
</table>

1. Medal totals are through 2004 Summer Olympics.

Reading to Learn

Mathematics, p. 171

ELL

Reading the Lesson

1. Complete the following sentence: A ratio is a comparison of two numbers by... division
2. Describe two ways to decide whether a sentence is a proportion. Express the ratios in simplest form to see if they are equal. Check to see whether the cross products are equal.

3. For each proportion, tell what the extremes are and what the means are.

4. A jet flying at a steady speed traveled 355 miles in 2 hours. If you double the proportion \( \frac{2}{x} = \frac{10}{5} \), what would the answer tell you about the jet?

Helping You Remember

5. Write how you would explain solving a proportion to a friend who missed Lesson 1-4.

Enrichment

Angles of a Triangle

So far, we have used the Triangle Sum Theorem to find the measures of the angles of a triangle. In this section, we will use the Triangle Sum Theorem to find the measures of the angles of a triangle.

For each of the triangles, write an equation and then solve for \( x \) (a tick mark on two or more sides of a triangle indicates that the sides have equal measures).

1. \( \angle x = 45^\circ \)
2. \( \angle x = 30^\circ \)
31. **WORK** Seth earns $152 in 4 days. At that rate, how many days will it take him to earn $532? 14 days

32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles? 1 1/2 h

33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint? 3 in.

34. **MODELS** A collector’s model racecar is scaled so that 1 inch on the model equals 6 1/4 feet on the actual car. If the model is 2 3/8 inch high, how high is the actual car? 41 1/4 ft

35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder? 18

36. **CRITICAL THINKING** Consider the proportion $a:b:c = 3:1:5$. What is the value of $2a + 3b$? (Hint: Choose different values of $a$, $b$, and $c$ for which the proportion is true and evaluate the expression.) 9

37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

How are ratios used in recipes?

Include the following in your answer:

- an explanation of how to use a proportion to determine how much honey is needed if you use 3 eggs, and
- a description of how to alter the recipe to get 5 servings.

38. Which ratio is not equal to 9/12? D

A 18/24 B 3/4 C 15/20 D 18/27

39. In the figure at the right, $x : y = 2 : 3$ and $y : z = 3 : 5$.

If $x = 10$, find the value of $z$. C

A 15 B 20 C 25 D 30

### Maintain Your Skills

#### Mixed Review

41. no solution

40. $8y - 10 = -3y + 2$ 41. $17 + 2n = 21 + 2n$ 42. $-7(d - 3) = -4$ 43. $5 - 9w = 23$ 44. $\frac{m}{5} + 6 = 31$ 45. $\frac{z - 7}{5} = -3$ 46. $(-7)(-6)$ 47. $\left( \frac{8}{9} \right)^{\frac{9}{8}}$ 48. $\left( \frac{3}{7} \right)^{\frac{3}{7}}$ 49. $(-0.075)(-5.5)$

46. $(-7)(-6)$ 47. $\left( \frac{8}{9} \right)^{\frac{9}{8}}$ 48. $\left( \frac{3}{7} \right)^{\frac{3}{7}}$ 49. $(-0.075)(-5.5)$ 50. $|\frac{33}{4}|$ 51. $\frac{77}{77}$ 52. $\frac{2.5}{2.5}$ 53. $|-0.85|$ 54. Sketch a reasonable graph for the temperature in the following statement.

In August, you enter a hot house and turn on the air conditioner. (Lesson 1-9)

54. Sketch a reasonable graph for the temperature in the following statement.

In August, you enter a hot house and turn on the air conditioner. (Lesson 1-9)

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Find each percent. (To review percents, see pages 802 and 803.)

55. Eighteen is what percent of 60? 30% 56. What percent of 14 is 4.34? 31%

57. Six is what percent of 15? 40% 58. What percent of 2 is 8? 400%

www.algebra1.com/self_check_quiz

### Online Lesson Plans

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. *Experience TODAY,* USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

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**Open-Ended Assessment**

**Modeling** Give students coins, paper clips, or other manipulatives and have them model a simple proportion such as $\frac{2}{3} = \frac{6}{9}$.

### Getting Ready for Lesson 3-7

**PREREQUISITE SKILL** Students will learn about percent of change in Lesson 3-7. Students use a proportion to find percent of change but need to understand how to find percents in order to set the proportion up correctly. Use Exercises 55–58 to determine your students’ familiarity with finding percents.

### Answers

2. A ratio is a comparison of two numbers and a proportion is an equation of two equal ratios.

3. Find the cross products and divide by the value of the variable.

37. Sample answer: Ratios are used to determine how much of each ingredient to use for a given number of servings. Answers should include the following.

- To determine how much honey is needed if you use 3 eggs, write and solve the proportion $2\frac{3}{4} = 3h$, where $h$ is the amount of honey.

- To alter the recipe to get 5 servings, multiply each amount by $1\frac{1}{4}$.
5-Minute Check

Transparency 3-7  Use as a quiz or review of Lesson 3-6.

Mathematical Background  notes are available for this lesson on p. 118D.

How can percents describe growth over time?

Ask students:

• What do the numbers 84, 171, and 285 represent on the graph? The number 84 represents the number of area codes in 1947, 171 was the number of area codes in 1996, and 285 was the number of area codes in 1999.

• Are these numbers percents? Why do you think so? No, these numbers are not percents because they are not followed by the percent sign.

• Do you think 171 is at least 100% more than 84? Explain your reasoning. 100% of 84 is 84. So, 84 + 84 is 168. Since 171 is greater than 168, then 171 must be at least 100% more than 84.

Vocabulary

• percent of change
• percent of increase
• percent of decrease

How can percents describe growth over time?

Phone companies began using area codes in 1947. The graph shows the number of area codes in use in different years. The growth in the number of area codes can be described by using a percent of change.

PERCENT OF CHANGE  When an increase or decrease is expressed as a percent, the percent is called the percent of change. If the new number is greater than the original number, the percent of change is a percent of increase. If the new number is less than the original, the percent of change is a percent of decrease.

Example 1  Find Percent of Change

State whether each percent of change is a percent of increase or a percent of decrease. Then find each percent of change.

a. original: 25
   new: 28
   Find the amount of change. Since the new amount is greater than the original, the percent of change is a percent of increase.
   \[ 28 - 25 = 3 \]
   Find the percent using the original number, 25, as the base.
   \[
   \frac{3}{25} = \frac{r}{100} \\
   300 = 25r \\
   300 = 25r \\
   12 = r
   \]
   The percent of increase is 12%.

b. original: 30
   new: 12
   The percent of change is a percent of decrease because the new amount is less than the original. Find the change.
   \[ 30 - 12 = 18 \]
   Find the percent using the original number, 30, as the base.
   \[
   \frac{18}{30} = \frac{r}{100} \\
   1800 = 30r \\
   1800 = 30r \\
   60 = r
   \]
   The percent of decrease is 60%.

Study Tip

Look Back
To review the percent proportion, see page 834.

Resource Manager

Workbook and Reproducible Masters

Chapter 3 Resource Masters
• Study Guide and Intervention, pp. 173–174
• Skills Practice, p. 175
• Practice, p. 176
• Reading to Learn Mathematics, p. 177
• Enrichment, p. 178
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Graphing Calculator and Spreadsheet Masters, p. 28
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5-Minute Check Transparency 3-7
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Technology
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Example 2 Find the Missing Value

**FOOTBALL** The field used by the National Football League (NFL) is 120 yards long. The length of the field used by the Canadian Football League (CFL) is 25% longer than the one used by the NFL. What is the length of the field used by the CFL?

Let \( \ell \) = the length of the CFL field. Since 25% is a percent of increase, the length of the NFL field is less than the length of the CFL field. Therefore, \( \ell - 120 \) represents the amount of change.

\[
\begin{align*}
\text{change} & \quad \ell - 120 \quad \frac{25}{100} \\
\text{original amount} & \quad 120 \quad 100 \\
(\ell - 120)(100) & = 120(25) \\
100\ell - 12,000 & = 3000 \\
100\ell & = 15,000 \\
\ell & = 150
\end{align*}
\]

The length of the field used by the CFL is 150 yards.

SOLVE PROBLEMS Two applications of percent of change are sales tax and discounts. Sales tax is a tax that is added to the cost of the item. It is an example of a percent of increase. Discount is the amount by which the regular price of an item is reduced. It is an example of a percent of decrease.

Example 3 Find Amount After Sales Tax

**SALES TAX** A concert ticket costs $45. If the sales tax is 6.25%, what is the total price of the ticket?

The tax is 6.25% of the price of the ticket.

\[
6.25\% \text{ of } 45 = 0.0625 \times 45 = 2.8125
\]

Round $2.8125 to $2.82 since tax is always rounded up to the nearest cent. Add this amount to the original price.

$45.00 + $2.82 = $47.82

The total price of the ticket is $47.82.

Example 4 Find Amount After Discount

**DISCOUNT** A sweater is on sale for 35% off the original price. If the original price of the sweater is $38, what is the discounted price?

The discount is 35% of the original price.

\[
35\% \text{ of } 38 = 0.35 \times 38 = 13.30
\]

Subtract $13.30 from the original price.

$38.00 - $13.30 = $24.70

The discounted price of the sweater is $24.70.

www.algebra1.com/extra_examples

Lesson 3-7 Percent of Change 161

**Differentiated Instruction**

**Naturalist** Have students make a list of three items for which they know the exact price. The items can be gifts they want to buy for themselves or everyday household items. Then have students calculate the price of each item if each price were discounted 15%.
1. **Concept Check**
   - **Percent of increase and percent of decrease are both percents of change. If the new number is greater than the original number, the percent of change is a percent of increase. If the new number is less than the original number, the percent of change is a percent of decrease.**

2. **OPEN ENDED**
   - Give a counterexample to the statement *The percent of change must always be less than 100%.* See margin.

3. **FIND THE ERROR**
   - Laura and Cory are writing proportions to find the percent of change if the original number is 20 and the new number is 30.
   - Laura: $\frac{30 - 20}{20} = \frac{10}{20} = \frac{r}{100}$
   - Cory: $\frac{30 - 20}{50} = \frac{10}{50} = \frac{r}{100}$
   - Who is correct? Explain your reasoning. Laura; Cory used the new number as the base instead of the original number.

**Guided Practice**

**GUIDED PRACTICE KEY**

<table>
<thead>
<tr>
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<th>Examples</th>
</tr>
</thead>
<tbody>
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<tr>
<td>8, 9</td>
<td>3</td>
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<tr>
<td>10, 11</td>
<td>4</td>
</tr>
</tbody>
</table>

**Application**

For Exercises 12 and 13, use the following information.

According to the Census Bureau, the average income of a person with a bachelor’s degree is $40,478, and the average income of a person with a high school diploma is $22,895.

12. Write an equation that could be used to find the percent of increase in average income for a person with a high school diploma to average income for a person with a bachelor’s degree.

13. What is the percent of increase? about 77%

\[
\frac{40,478 - 22,895}{22,895} = \frac{x}{100}
\]

**Answers**

2. Sample answer: If the original number is 10 and the new number is 30, the percent proportion is \( \frac{30 - 10}{10} = \frac{r}{100} \) and the percent of change is 200%, which is greater than 100%.

49. \( x \% \) of \( y \Rightarrow \frac{x}{100} = \frac{y}{P} \) or \( P = \frac{xy}{100} \)

\( y \% \) of \( x \Rightarrow \frac{y}{100} = \frac{P}{x} \) or \( P = \frac{xy}{100} \)
26. **THEME PARKS**  In 1990, 253 million people visited theme parks in the United States. In 2000, the number of visitors increased to 317 million people. What was the percent of increase? **about 25%**

27. **MILITARY**  In 1987, the United States had 2 million active-duty military personnel. By 2000, there were only 1.4 million active-duty military personnel. What was the percent of decrease? **30%**

28. The percent of increase is 16%. If the new number is 522, find the original number. **450**

29. **FOOD**  In order for a food to be marked “reduced fat,” it must have at least 25% less fat than the same full-fat food. If one ounce of reduced fat chips has 6 grams of fat, what is the least amount of fat in one ounce of regular chips? **8 g**

30. **TECHNOLOGY**  From January, 1996, to January, 2001, the number of internet hosts increased by 105%. There were 109.6 million internet hosts in January, 2001, Find the number of internet hosts in January, 1996. **9.5 million internet hosts**

31. Umbrella: $14.00
   - tax: 5.5% **$14.77**
   - total: $14.77

32. Backpack: $35.00
   - tax: 7% **$37.45**
   - total: $37.45

33. Candle: $7.50
   - tax: 5.75% **$7.93**
   - total: $7.93

34. Sandals: $29.99
   - tax: 5.75% **$31.71**
   - total: $31.71

35. Clock radio: $39.99
   - tax: 6% **$42.46**
   - total: $42.46

36. Watch: $73.55
   - tax: 6% **$77.50**
   - total: $77.50

37. Shirt: $45.00
   - discount: 40% **$27.00**
   - discounted price: $27.00

38. Socks: $6.00
   - discount: 20% **$4.80**
   - discounted price: $4.80

39. Watch: $37.55
   - discount: 35% **$24.41**
   - discounted price: $24.41

40. Gloves: $24.25
   - discount: 33% **$16.25**
   - discounted price: $16.25

41. Suit: $175.95
   - discount: 45% **$96.77**
   - discounted price: $96.77

42. Coat: $79.99
   - discount: 30% **$55.99**
   - discounted price: $55.99

43. Lamp: $120.00
   - discount: 20% **$96.00**
   - discounted price: $96.00

44. Dress: $70.00
   - discount: 30% **$49.00**
   - discounted price: $49.00

45. Camera: $58.00
   - discount: 25% **$43.50**
   - discounted price: $43.50

46. China: about 1.52 billion people; India: about 1.53 billion people; United States: about 0.39 billion people

47. Which of these three countries is projected to be the most populous in 2050? **India**

48. **RESEARCH**  Use the Internet or other reference to find the tuition for the last several years at a college of your choice. Find the percent of change for the tuition during these years. Predict the tuition for the year you plan to graduate from high school. **See students work**

49. **CRITICAL THINKING**  Are the following expressions sometimes, always, or never equal? Explain your reasoning. **Always; see margin for explanation.**

50. **HELPING YOU REMEMBER**  A military career can involve several years at a college of your choice. Find the percent of change for the percent of increase?

### Enrichment, p. 178

**Using Percent**

Use what you have learned about percent to solve each problem.

- **Part A**
  - **Problem:**
    - A TV series has both a "rating" of 11 and a "share" rating. The rating is the percentage of the nation’s total TV households that were tuned in to the show. The share is the percentage of homes with TV sets tuned to the show. If we find out, let F = the number of TV households and a = the number of TV households tuned to the show.
    - The F rating is calculated as a/F.
    - The share is calculated as a/F.
  - **Example:**
    - If 11.1 million people watched the TV show, then 11.1 million/90 million = 0.123.
    - 12.3%

- **Part B**
  - **Problem:**
    - ABC’s sitcom “Family Matters” had a F rating of 5.5 and a share of 35.
    - **Question:**
      - What is the percentage of households tuned to the show?
      - **Answer:**
        - 35%

The percent of the TV households tuned to the sitcom was 35%

### Reading to Learn Mathematics, p. 177

**Pre-Activity**

- **How can percents describe growth over time?**
- **Read the introduction to Lesson 3-7 at the top of page 505 in your textbook.
- **How many area codes were in use in 1960? 9.
- **How many area codes were in use in 1999? 281 area codes**

**Reading the Lesson**

1. If you use the original amount — new amount to find the percent of change, then the problem involves a percent of increase (over the original amount).
2. If you use the original amount — new amount to find the percent of change, then the problem involves a percent of decrease (under the original amount).
4 Assess

Open-Ended Assessment
Speaking Give students pairs of numbers. Tell which number is the original. Have students determine whether the percent of increase or the percent of decrease is needed to find the other number.

Getting Ready for Lesson 3–8
PREREQUISITE SKILL Students will solve equations and formulas for a given variable in Lesson 3–8. This requires algebraic manipulation, which they used in solving equations in one variable. Use Exercises 66–71 to determine your students’ familiarity with solving equations.

Assessment Options
Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 3–4 through 3–7. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 3–6 and 3–7) is available on p. 206 of the Chapter 3 Resource Masters.

Answer
50. Find the amount of change and express this change as a percent of the original number. Answers should include the following.
- To find the percent of increase, first find the amount of increase. Then find what percent the amount of increase is of the original number.
- The percent of increase from 1996 to 1999 is about 67%.
- An increase of 100 is a very large increase if the original number is 50, but a very small increase if the original number is 100,000. The percent of change will indicate whether the change is large or small relative to the original.

51. The number of students at Franklin High School increased from 840 to 910 over a 5-year period. Which proportion represents the percent of change?

52. The list price of a television is $249.00. If it is on sale for 30% off the list price, what is the sale price of the television?

Maintain Your Skills

Mixed Review Solve each proportion. (Lesson 3-6)
53. \( \frac{a}{45} = \frac{3}{15} \) 9
54. \( \frac{2}{3} = \frac{8}{d} \) 12
55. \( \frac{5.22}{13.92} = \frac{t}{48} \) 18

Solve each equation. Then check your solution. (Lesson 3-5)
56. \( 6n + 3 = -3 \) 1 57. \( 7 + 5c = -23 \) 22 58. \( 18 = 4n - 2 \) 5

Find each quotient. (Lesson 2-4)
59. \( \frac{2}{5} \div 4 \) 10 60. \( \frac{-4}{5} \div \frac{2}{3} \) 1.1 61. \( \frac{-1}{9} \div \left( \frac{3}{4} \right) \) 4 27

State whether each equation is true or false for the value of the variable given. (Lesson 1-3)
62. \( a^2 + 5 = 17 \), \( a = 3 \) true 63. \( 2v^2 + v = 65 \), \( v = 5 \) false 64. \( 8y - y^2 = y + 10 \), \( y = 4 \) false 65. \( 16p - p = 15p \), \( p = 2.5 \) true

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation. Then check your solution. (To review solving equations, see Lesson 3-5.)
66. \(-43 - 3t = 2 - 6t \) 15 67. \( 7y + 7 = 3y - 5 \) 12 68. \( 7(d - 3) = 2 \) 5 69. \( 6(p + 3) = 4(p - 1) \) 11 70. \(-5 = 4 - 2(a - 5) \) 9.5 71. \( 8x - 4 = -10x + 50 \) 3

Practice Quiz 2

Solve each equation. Then check your solution. (Lessons 3-4 and 3-5)
1. \(-3x - 7 = 18 \) 9 2. \( 5 = \frac{m - 5}{4} \) 25 3. \(-3 + 5 = 11 \) 1.5
4. \( 4d - 6 = 3d + 9 \) 7.5 5. \( 7 + 2(w + 1) = 2w + 9 \) all numbers 6. \(-8(4 + 9r) = 7(-2 - 11r) \) 3.6

Solve each proportion. (Lesson 3-6)
7. \( \frac{2}{10} = \frac{1}{a} \) 5 8. \( \frac{3}{5} = \frac{24}{x} \) 40 9. \( \frac{6}{4} = \frac{y + 5}{8} \) 5

10. POSTAGE In 1975, the cost of a first-class stamp was 10¢. In 2001, the cost of a first-class stamp became 34¢. What is the percent of increase in the price of a stamp? (Lesson 3-7) 248%

Teacher to Teacher

Barbara Szymczak Bayonne H.S., Bayonne, NJ

“When teaching percents, I have students cut out sales advertisements and make posters showing savings. They also cut out car ads to compute interest they would pay and compare companies for the best deals.”
Sentence Method and Proportion Method

Recall that you can solve percent problems using two different methods. With either method, it is helpful to use “clue” words such as is and of. In the sentence method, is means equals and of means multiply. With the proportion method, the “clue” words indicate where to place the numbers in the proportion.

Sentence Method
15% of 40 is what number?

0.15 \cdot 40 = ?

Proportion Method
15% of 40 is what number?

\[
\text{Proportion method: } \frac{(is) \ P}{(of) \ B} = \frac{R(\text{percent})}{100} \rightarrow \frac{P}{40} = \frac{15}{100}
\]

You can use the proportion method to solve percent of change problems. In this case, use the proportion \( \text{difference} \over \text{original} = \frac{\%}{100} \). When reading a percent of change problem, or any other word problem, look for the important numerical information.

Example
In chemistry class, Kishi heated 20 milliliters of water. She let the water boil for 10 minutes. Afterward, only 17 milliliters of water remained, due to evaporation. What is the percent of decrease in the amount of water?

\[
\frac{\text{difference}}{\text{original}} = \frac{\%}{100} \rightarrow \frac{20 - 17}{20} = \frac{r}{100} \text{ Percent proportion}
\]

\[
\frac{3}{20} = \frac{r}{100} \text{ Simplify.}
\]

\[
3(100) = 20r \quad \text{Find the cross products.}
\]

\[
300 = 20r \quad \text{Simplify.}
\]

\[
300 = 20r \rightarrow 20 \quad \text{Divide each side by 20.}
\]

\[
15 = r \quad \text{Simplify.}
\]

There was a 15% decrease in the amount of water.

1–3. See margin for original number, amount of change, and percent proportion.

Reading to Learn
Give the original number and the amount of change. Then write and solve a percent proportion.

1. Monsa needed to lose weight for wrestling. At the start of the season, he weighed 166 pounds. By the end of the season, he weighed 158 pounds. What is the percent of decrease in Monsa’s weight? about 5%

2. On Carla’s last Algebra test, she scored 94 points out of 100. On her first Algebra test, she scored 70 points out of 100. What is the percent of increase in her score? about 25%

3. In a catalog distribution center, workers processed an average of 12 orders per hour. After a reward incentive was offered, workers averaged 18 orders per hour. What is the percent of increase in production? 50%

Answers

1. original number: 166 lb; amount of change: 166 – 158 or 8 lb; \( \frac{8}{166} = \frac{r}{100} \)

2. original number: 75 points; amount of change: 94 – 75 or 19 points; \( \frac{19}{75} = \frac{r}{100} \)

3. original number: 12 orders; amount of change: 18 – 12 or 6 orders; \( \frac{6}{12} = \frac{r}{100} \)
Solving Equations and Formulas

What You’ll Learn

• Solve equations for given variables.
• Use formulas to solve real-world problems.

Vocabulary

• dimensional analysis

How are equations used to design roller coasters?

Ron Toomer designs roller coasters, including the Magnum XL-200. This roller coaster starts with a vertical drop of 195 feet and then ascends a second shorter hill. Suppose when designing this coaster, Mr. Toomer decided he wanted to adjust the height of the second hill so that the coaster would have a speed of 49 feet per second when it reached its top.

If we ignore friction, the equation \( g(195 - h) = \frac{1}{2}v^2 \) can be used to find the height of the second hill. In this equation, \( g \) represents the force of gravity (32 feet per second squared), \( h \) is the height of the second hill, and \( v \) is the velocity of the coaster when it reaches the top of the second hill.

Example 1 Solve an Equation for a Specific Variable

Solve 3x – 4y = 7 for y.

\[
3x - 4y = 7 \\
3x - 4y - 3x = 7 - 3x \\
-4y = 7 - 3x \\
\frac{-4y}{-4} = \frac{7 - 3x}{-4} \\
y = \frac{7 - 3x}{4} \quad \text{or} \quad \frac{3x - 7}{4}
\]

The value of \( y \) is \( \frac{3x - 7}{4} \).

It is sometimes helpful to use the Distributive Property to isolate the variable for which you are solving an equation or formula.

Resource Manager

Workbook and Reproducible Masters

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• Reading to Learn Mathematics, p. 183
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Prerequisite Skills Workbook, pp. 81–82

Transparencies

5-Minute Check Transparency 3-8
Answer Key Transparencies

Technology

AlgePASS: Tutorial Plus, Lessons 7, 8
Interactive Chalkboard
**Example 2** Solve an Equation for a Specific Variable

Solve \(2m - t = sm + 5\) for \(m\).

\[
\begin{align*}
2m - t &= sm + 5 \\
2m - t - sm &= sm + 5 - sm \\
2m - t - sm &= 5 \\
2m - t &= 5 + t \\
m(2 - s) &= 5 + t \\
\frac{m(2 - s)}{2 - s} &= \frac{5 + t}{2 - s} \\
m &= \frac{5 + t}{2 - s} \\
\end{align*}
\]

The value of \(m\) is \(\frac{5 + t}{2 - s}\). Since division by 0 is undefined, \(2 - s \neq 0\) or \(s \neq 2\).

**Example 3** Use a Formula to Solve Problems

- **WEATHER** Use the information about the Kansas City hailstorm at the left. The formula for the circumference of a circle is \(C = 2\pi r\), where \(C\) represents circumference and \(r\) represents radius.

  a. Solve the formula for \(r\).

  \[
  \begin{align*}
  C &= 2\pi r \\
  \frac{C}{2\pi} &= \frac{2\pi r}{2\pi} \\
  \frac{C}{2\pi} &= r \\
  \end{align*}
  \]

  b. Find the radius of one of the large hailstones that fell on Kansas City in 1898.

  \[
  \begin{align*}
  \frac{C}{2\pi} &= r \\
  9.5 &= \frac{2\pi r}{2\pi} \\
  9.5 &= r \\
  2\pi &= r \\
  \frac{2\pi}{r} &= r \\
  \end{align*}
  \]

  The largest hailstones had a radius of about 1.5 inches.

When using formulas, you may want to use dimensional analysis. **Dimensional analysis** is the process of carrying units throughout a computation.

**Example 4** Use Dimensional Analysis

- **PHYSICAL SCIENCE** The formula \(s = \frac{1}{2}at^2\) represents the distance \(s\) that a free-falling object will fall near a planet or the moon in a given time \(t\). In the formula, \(a\) represents the acceleration due to gravity.

  a. Solve the formula for \(a\).

  \[
  \begin{align*}
  s &= \frac{1}{2}at^2 \\
  \frac{2s}{t^2} &= \frac{\frac{1}{2}at^2}{t^2} \\
  2a &= \frac{2s}{t^2} \\
  a &= \frac{2s}{t^2} \\
  \end{align*}
  \]

**In-Class Examples**

1. Solve \(5b + 12c = 9\) for \(b\).

   \[
   b = \frac{9 - 12c}{5}
   \]

2. Solve \(7x - 2z = 4 - xy\) for \(x\).

   \[
   x = \frac{4 + 2z}{7 + y}
   \]

**USE FORMULAS**

3. **FUEL ECONOMY** A car’s fuel economy \(E\) (miles per gallon) is given by the formula \(E = \frac{m}{g}\), where \(m\) is the number of miles driven and \(g\) is the number of gallons of fuel used.

   a. Solve the formula for \(m\).

   \[
   m = Eg
   \]

   b. If Claudia’s car has an average fuel consumption of 30 miles per gallon and she used 9.5 gallons, how far did she drive? 285 mi.

4. **GEOMETRY** The formula for the volume of a cylinder is \(V = \pi r^2 h\), where \(r\) is the radius of the cylinder and \(h\) is the height.

   a. Solve the formula for \(h\).

   \[
   h = \frac{V}{\pi r^2}
   \]

   b. What is the height of a cylindrical swimming pool that has a radius of 12 feet and a volume of 1810 cubic feet? about 4 ft.
Check for Understanding

Concept Check
1. List the steps you would use to solve $ax - y = az + w$ for $a$. See margin.
2. Describe the possible values of $t$ if $s = \frac{t}{1 - 2}$. $t$ can be any number except 2.
3. OPEN ENDED Write a formula for $A$, the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than $A$.

Guided Practice
Solve each equation or formula for the variable specified.

4. $-3x + b = 6x$, for $x$ $x = \frac{b}{9}$
5. $-5x + y = -54$, for $a$ $a = \frac{54 + y}{5}$
6. $4x + b = 2x + c$, for $z$ $z = \frac{c - b}{p}$
7. $y + \frac{a}{3} = c$, for $y$ $y = \frac{3c - a}{2}$
8. $p = a(b + c)$, for $a$ $a = \frac{p}{b + c}$
9. $mw - t = 2w + 5$, for $w$ $w = \frac{5 + t}{m - 2}$

GEOMETRY For Exercises 10–12, use the formula for the area of a triangle.

10. Find the area of a triangle with a base of 18 feet and a height of 7 feet. $63 \text{ ft}^2$
11. Solve the formula for $h$. $h = \frac{2A}{b}$
12. What is the height of a triangle with area of 28 square feet and base of 8 feet? 7 ft

Practice and Apply

Homework Help
See page 827.

Extra Practice

13. $5x + h = g$, for $x$ $g = \frac{-h}{4}$
14. $8t - r = 12t$, for $t$ $t = \frac{-r}{4}$
15. $y = mx + b$, for $m$ $m = \frac{y - b}{x}$
16. $v = r + at$, for $a$ $a = \frac{v - r}{t}$
17. $3y + z = am - 4y$, for $y$ $y = \frac{am - z}{7}$
18. $9a - 2b = c + 4a$, for $a$ $a = \frac{2b + c}{5}$
19. $km + 5x = 6y$, for $m$ $m = \frac{6y - 5x}{k}$
20. $4b - 5 = -t$, for $b$ $b = \frac{5 - t}{4}$
21. $\frac{3ax - n}{5} = -4$, for $x$ $x = \frac{n - 20}{3a}$
22. $\frac{5x + y}{a} = 2$, for $a$ $a = \frac{5x + y}{2}$
23. $\frac{by + 2}{3} = c$, for $y$ $y = \frac{3c - 2}{b}$
24. $\frac{6c - t}{7} = b$, for $c$ $c = \frac{7b + t}{6}$
25. $c = \frac{4}{3}y + b$, for $y$ $y = \frac{3}{4}(c - b)$
26. $\frac{3}{5}m + a = b$, for $m$ $m = \frac{5}{3}(b - a)$

27. $S = \frac{1}{2}(A + t)$, for $A$ $A = \frac{2S - nt}{n}$
28. $p(t + 1) = -2$, for $t$ $t = \frac{-2 - p}{p}$
29. $at + b = ar - c$, for $a$ $a = \frac{c + b}{r - t}$
30. $2x - m = 5 - gh$, for $g$ $g = \frac{5 + m}{2 + h}$

About the Exercises...
Organization by Objective
• Solve for Variables: 13–33
• Use Formulas: 34–41

Odd/Even Assignments
Exercises 13–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Average: 13–29 odd, 34–39, 42–66
Advanced: 14–30 even, 31–33, 38–60 (optional: 61–66)

Practice and Apply

Answer
1. (1) Subtract $az$ from each side.
(2) Add $y$ to each side.
(3) Use the Distributive Property to write $ax - az$ as $a(x - z)$.
(4) Divide each side by $x - z$.

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Write Watch Out! at the top of the chalkboard before students begin working the Check for Understanding problems. Invite students to compile a collective list of mistakes to watch out for as they work these problems. For each mistake, have students suggest a way they could work differently to avoid the mistake.
Write an equation and solve for the variable specified.

31. \( t - 5 = r + 6; \quad t = r + 11 \)
32. Five minus twice a number \( p \) equals six times another number \( q \) plus one. Solve for \( p \).
\[ 5 - 2p = 6q + 1; \quad p = 2 - 3q \]
33. Five eighths of a number \( x \) is three more than one half of another number \( y \).
\[ \frac{5}{8}x = \frac{1}{2}y + 3; \quad y = \frac{5}{4}x - 6 \]

GEOMETRY
For Exercises 34 and 35, use the formula for the area of a trapezoid.

34. Solve the formula for \( h \).
\[ h = \frac{2A}{b + a} \]
35. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters? 6 m

36. Solve for \( c \).
\[ e = \frac{w - sm}{h} \]
37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make? 3 errors

FLOORING
For Exercises 38 and 39, use the following information.
The formula \( P = \frac{12W}{H^2} \) is often used by placement services to find keyboarding speeds. In the formula, \( s \) represents the speed in words per minute, \( w \) represents the number of words typed, \( e \) represents the number of errors, and \( m \) represents the number of minutes typed.

38. Solve the formula for \( W \).
\[ W = \frac{H^2P}{12} \]
39. Find the weight of the person if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch. 225 lb

ROCKETRY
In the book October Sky, high school students were experimenting with different rocket designs. One formula they used was \( R = \frac{S + P}{S + P} \), which relates the mass ratio \( R \) of a rocket to the mass of the structure \( S \), the mass of the fuel \( F \), and the mass of the payload \( P \). The students needed to determine how much fuel to load in the rocket. How much fuel should be loaded in a rocket whose basic structure and payload each have a mass of 900 grams, if the mass ratio is to be 6? 9000 g

PACKAGING
The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be?
\( \text{Volume} = \pi r^2 h \)
about 17.4 cm

41. Enrichment, p. 184
Dr. Bernardou Houussy

Enrichment, p. 184
Dr. Bernardou Houussy

Reading to Learn Mathematics, p. 183

Skills Practice, p. 181 and Practice, p. 182 (shown)

Solve each equation or formula for the variable specified.

ELL

WORK
For Exercises 36 and 37, use the following information.
The formula \( s = \frac{w - e}{m} \) is often used by placement services to find keyboarding speeds. In the formula, \( s \) represents the speed in words per minute, \( w \) represents the number of words typed, \( e \) represents the number of errors, and \( m \) represents the number of minutes typed.

36. Solve the formula for \( e \).
\[ e = \frac{w - sm}{h} \]
37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make? 3 errors

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\( \text{Volume} = \pi r^2 h \)
about 17.4 cm

www.algebra1.com/self_check_quiz
42. **CRITICAL THINKING** Write a formula for the area of the arrow. $A = \frac{1}{2}bh$

43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How are equations used to design roller coasters?
Include the following in your answer:
- a list of steps you could use to solve the equation for $h$, and
- the height of the second hill of the roller coaster.

44. If $2x + y = 5$, what is the value of $4x$? **B**
   - A) $10 - y$
   - B) $10 - 2y$
   - C) $\frac{5 - y}{2}$
   - D) $\frac{10 - y}{2}$

45. What is the area of the triangle? **C**
   - A) 23 m$^2$
   - B) 28 m$^2$
   - C) 56 m$^2$
   - D) 112 m$^2$

---

**Open-Ended Assessment**

**Writing** Have students pick a formula that was not used in this lesson, perhaps from a science class, and explain the variables in the formula and what the formula is used to find. Then have students solve the formula for the different variables.

---

**Getting Ready for Lesson 3-9**

**PREREQUISITE SKILL** Students will learn about weighted averages in Lesson 3-9. Weighted average problems frequently require students to use the Distributive Property to simplify equations containing expressions in parentheses. Use Exercises 61–66 to determine your students’ familiarity with the Distributive Property.

---

**Answer**

43. Equations from physics can be used to determine the height needed to produce the desired results. Answers should include the following.

- Use the following steps to solve for $h$.
  1. Use the Distributive Property to write the equation in the form $195g - hg = \frac{1}{2}mv^2$.
  2. Subtract $195g$ from each side.
  3. Divide each side by $g$.
- The second hill should be 157 ft.
**Weighted Averages**

### What You'll Learn
- Solve mixture problems.
- Solve uniform motion problems.

### Vocabulary
- weighted average
- mixture problem
- uniform motion problem

### How are scores calculated in a figure skating competition?
In individual figure skating competitions, the score for the long program is worth twice the score for the short program. Suppose Olympic gold medal winner Ilia Kulik scores 5.5 in the short program and 5.8 in the long program at a competition. His final score is determined using a weighted average.

\[ \frac{5.5(1) + 5.8(2)}{1 + 2} = \frac{5.5 + 11.6}{3} = \frac{17.1}{3} = 5.7 \]

His final score would be 5.7.

### Mixture Problems
Ilia Kulik’s average score is an example of a weighted average. The weighted average \( M \) of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages.

### Example 1 Solve a Mixture Problem with Prices

**Trail Mix** Assorted dried fruit sells for $5.50 per pound. How many pounds of mixed nuts selling for $4.75 per pound should be mixed with 10 pounds of dried fruit to obtain a trail mix that sells for $4.95 per pound?

Let \( w \) = the number of pounds of mixed nuts in the mixture. Make a table.

<table>
<thead>
<tr>
<th>Units (lb)</th>
<th>Price per Unit (lb)</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dried Fruit</td>
<td>10</td>
<td>5.50(10)</td>
</tr>
<tr>
<td>Mixed Nuts</td>
<td>( w )</td>
<td>4.75( w )</td>
</tr>
<tr>
<td>Trail Mix</td>
<td>10 + ( w )</td>
<td>4.95(10 + ( w ))</td>
</tr>
</tbody>
</table>

Price of dried fruit plus price of nuts equals price of trail mix.

\[ 5.50(10) + 4.75\( w \) = 4.95(10 + \( w \)) \]

Original equation

\[ 55.00 + 4.75\( w \) = 49.50 + 4.95\( w \) \]

Distributive Property

\[ 55.00 + 4.75\( w \) - 4.75\( w \) = 49.50 + 4.95\( w \) - 4.75\( w \) \]

Subtract 4.75\( w \) from each side.

\[ 55.00 = 49.50 + 0.20\( w \) \]

Simplify.

\[ 55.00 - 49.50 = 49.50 + 0.20\( w \) - 49.50 \]

Subtract 49.50 from each side.

\[ 5.50 = 0.20\( w \) \]

Simplify.

\[ \frac{5.50}{0.20} = \frac{0.20\( w \)}{0.20} \]

Divide each side by 0.20.

\[ 27.5 = \( w \) \]

Simplify.

27.5 pounds of nuts should be mixed with 10 pounds of dried fruit.

---

**Resource Manager**

**Workbook and Reproducible Masters**
- Chapter 3 Resource Masters
  - Study Guide and Intervention, pp. 185–186
  - Skills Practice, p. 187
  - Practice, p. 188
  - Reading to Learn Mathematics, p. 189
  - Enrichment, p. 190
  - Assessment, p. 206

**Parent and Student Study Guide Workbook**, p. 27

**Technology**
- Interactive Chalkboard

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**Focus**

**5-Minute Check Transparency 3-9** Use as a quiz or review of Lesson 3-8.

**Mathematical Background** notes are available for this lesson on p. 118D.

**How are scores calculated in a figure skating competition?**
Ask students:
- How would you find the unweighted average of the two scores? Add the two scores and divide the sum by the number of scores, which is two.
- What is the unweighted average of the two scores? 5.65
- How is the weighted average similar to adding another score? Since the score for the long program is counted twice, and the sum is divided by three, it is as if a third score has been added before the average is taken.
- Did the weighted average help or hurt Ilia Kulik’s score? Would this always be the case? Because Ilia Kulik scored higher in the long program than in the short program, the weighted average made his average score higher. If he had scored lower in the long program, then the weighted average would have hurt his score. If he scored the same in both programs, the weighted average would have had no effect.
MIXTURE PROBLEMS

Teaching Tip Explain to students that the object of this problem is to come up with a mixture of higher-priced dried fruit and cheaper nuts that has an average price of $4.95 per pound.

1 PETS Jeri likes to feed her cat gourmet cat food that costs $1.75 per pound. However, food at that price is too expensive so she combines it with cheaper cat food that costs $0.50 per pound. How many pounds of cheaper food should Jeri buy to go with 5 pounds of gourmet food, if she wants the average price to be $1.00 per pound? Jeri should buy 7.5 lb of cheaper food.

2 AUTO MAINTENANCE To provide protection against freezing, a car’s radiator should contain a solution of 50% antifreeze. Darryl has 2 gallons of a 35% antifreeze solution. How many gallons of 100% antifreeze should Darryl add to his solution to produce a solution of 50% antifreeze? Darryl should add 0.6 gal of 100% antifreeze to the solution.

Teaching Tip Once students have learned the concept of weighted average, challenge them to describe a weighted average in terms of weights on a balance. How do the weights help to “tip” the balance?

Example 2 Solve a Mixture Problem with Percents

SCIENCE A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Let $x$ = the amount of 60% solution to be added. Make a table.

<table>
<thead>
<tr>
<th>Amount of Copper Sulfate</th>
<th>Amount of Solution (mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% Solution</td>
<td>40</td>
</tr>
<tr>
<td>60% Solution</td>
<td>$x$</td>
</tr>
<tr>
<td>30% Solution</td>
<td>$40 + x$</td>
</tr>
</tbody>
</table>

Write and solve an equation using the information in the table.

\[
0.25(40) + 0.60x = 0.30(40 + x) \quad \text{Original equation}
\]
\[
10 + 0.60x = 12 + 0.30x \quad \text{Distributive Property}
\]
\[
10 + 0.60x - 0.30x = 12 + 0.30x - 0.30x \quad \text{Subtract 0.30x from each side.}
\]
\[
10 + 0.30x = 12 \quad \text{Simplify.}
\]
\[
10 + 0.30x - 10 = 12 - 10 \quad \text{Subtract 10 from each side.}
\]
\[
0.30x = 2 \quad \text{Simplify.}
\]
\[
\frac{0.30x}{0.30} = \frac{2}{0.30} \quad \text{Divide each side by 0.30.}
\]
\[
x = 6.67 \quad \text{Simplify.}
\]

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution.

Example 3 Solve for Average Speed

TRAVEL On Alberto’s drive to his aunt’s house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite $d = rt$ as $r = \frac{d}{t}$.

Going
\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{1 \text{ hour}} = 45 \text{ miles per hour}
\]

Returning
\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{2 \text{ hours}} = 22.5 \text{ miles per hour}
\]

Sometimes mixture problems are expressed in terms of percents.

Uniform motion problems are another application of weighted averages. Uniform motion problems are problems where an object moves at a certain speed, or rate. The formula $d = rt$ is used to solve these problems. In the formula, $d$ represents distance, $r$ represents rate, and $t$ represents time.

Example 3 Solve for Average Speed

TRAVEL On Alberto’s drive to his aunt’s house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

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Going
\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{1 \text{ hour}} = 45 \text{ miles per hour}
\]

Returning
\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{2 \text{ hours}} = 22.5 \text{ miles per hour}
\]
You may think that the average speed of the trip would be $\frac{45 + 22.5}{2}$ or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

**Round Trip**

$m = \frac{45(1) + 22.5(2)}{1 + 2}$  Definition of weighted average

$= \frac{90}{3}$ or 30  Simplify.

Alberto’s average speed was 30 miles per hour.

Sometimes a table is useful in solving uniform motion problems.

### Example 4  Solve a Problem Involving Speeds of Two Vehicles

**SAFETY**  Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals 1000 − 440 or 560 feet.

Let $t =$ the number of seconds until the driver can hear the siren.

Make a table of the information.

<table>
<thead>
<tr>
<th></th>
<th>$r$</th>
<th>$t$</th>
<th>$d = rt$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>44</td>
<td>$t$</td>
<td>44$t$</td>
</tr>
<tr>
<td>Emergency Squad</td>
<td>74</td>
<td>$t$</td>
<td>74$t$</td>
</tr>
</tbody>
</table>

Write an equation.

Distance traveled by car plus distance traveled by emergency vehicle equals 560 feet.

$44t + 74t = 560$

Solve the equation.

$118t = 560$

$t = 4.75$  Round to the nearest hundredth.

The driver of the car will hear the siren in about 4.75 seconds.
3 Practice/Apply

Study Notebook
Have students—
• complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 9.
• include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...
Organization by Objective
• Mixture Problems: 11–18, 22–25, 27–29, 33
• Uniform Motion Problems: 19–21, 26, 30–32, 34

Alert! Exercise 35 includes an Internet research extension question.

Assignment Guide
Basic: 11–14, 19–21, 23–31 odd, 36–50
Average: 11–14, 19–21, 23–35 odd, 36–50
Advanced: 15–21, 22–32 even, 33–50

Guided Practice

1. OPEN ENDED Give a real-world example of a weighted average.
2. Write the formula used to solve uniform motion problems and tell what each letter represents. \( d = rt; d = \text{distance}, \ r = \text{rate}, \ t = \text{time} \)
3. Make a table that can be used to solve the following problem.
Lakeisha has $2.55 in dimes and quarters. She has 8 more dimes than quarters. How many quarters does she have? See margin.

Extra Practice

For Exercises 11–14, use the following information.
Cookies Inc. sells peanut butter cookies for $6.50 per dozen and chocolate chip cookies for $9.00 per dozen. Yesterday, they sold 85 dozen more peanut butter cookies than chocolate chip cookies. The total sales for both types of cookies were $4055.50. Let \( p \) represent the number of dozens of peanut butter cookies sold.

11. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Dozens</th>
<th>Price per Dozen</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut Butter Cookies</td>
<td>( p )</td>
<td>$6.50</td>
</tr>
<tr>
<td>Chocolate Chip Cookies</td>
<td>( p - 85 )</td>
<td>$9.00</td>
</tr>
</tbody>
</table>

12. Write an equation to represent the problem. \( 6.50p + 9.00(p - 85) = 4055.50 \)
13. How many dozen peanut butter cookies were sold? 311 doz
14. How many dozen chocolate chip cookies were sold? 226 doz

Answer

3. Make a table that can be used to solve the following problem.

<table>
<thead>
<tr>
<th>Number of Coins</th>
<th>Value of Each Coin</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimes ( d )</td>
<td>( $0.10 )</td>
<td>( 0.10d )</td>
</tr>
<tr>
<td>Quarters ( d - 8 )</td>
<td>( $0.25 )</td>
<td>( 0.25(d - 8) )</td>
</tr>
</tbody>
</table>
**METALS**  For Exercises 15–18, use the following information.
In 2000, the international price of gold was $270 per ounce, and the international price of silver was $5 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth $164 per ounce. Let \( g \) represent the amount of gold used in the alloy.

15. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th></th>
<th>Number of Ounces</th>
<th>Price per Ounce</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>( g )</td>
<td>$270</td>
<td>( 270g )</td>
</tr>
<tr>
<td>Silver</td>
<td>( 15 - g )</td>
<td>$5</td>
<td>( 5(15 - g) )</td>
</tr>
<tr>
<td>Alloy</td>
<td>15</td>
<td>$164</td>
<td>( 164(15) )</td>
</tr>
</tbody>
</table>

16. Write an equation to represent the problem. \( 270g + 5(15 - g) = 164(15) \)

17. How much gold was used in the alloy? 9 oz

18. How much silver was used in the alloy? 6 oz

**TRAVEL**  For Exercises 19–21, use the following information.
Two trains leave Pittsburgh at the same time, one traveling east and the other traveling west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let \( t \) represent the amount of time since their departure.

19. Copy and complete the table representing the situation.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastbound Train</td>
<td>40</td>
<td>( t )</td>
<td>40( t )</td>
</tr>
<tr>
<td>Westbound Train</td>
<td>30</td>
<td>( t )</td>
<td>30( t )</td>
</tr>
</tbody>
</table>

20. Write an equation that could be used to determine when the trains will be 245 miles apart. \( 40t + 30t = 245 \)

21. In how many hours will the trains be 245 miles apart? \( 3 \frac{1}{2} \) h

22. **FUND-RAISING**  The Madison High School marching band sold gift wrap. The gift wrap in solid colors sold for $4.00 per roll, and the print gift wrap sold for $6.00 per roll. The total number of rolls sold was 480, and the total amount of money collected was $2340. How many rolls of each kind of gift wrap were sold? \( 270 \) rolls of solid wrap, \( 210 \) rolls of print wrap

23. **COFFEE**  Charley Baroni owns a specialty coffee store. He wants to create a special mix using two coffees, one priced at $6.40 per pound and the other priced at $7.28 per pound. How many pounds of the $7.28 coffee should he mix with 9 pounds of the $6.40 coffee to sell the mixture for $6.95 per pound? 15 lb

24. **FOOD**  Refer to the graphic at the right.
How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat? 10 gal of cream, 25 gal of 2% milk

25. **METALS**  An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each alloy should be used to make 1000 grams of an alloy that is 45% copper? 200 g of 25% alloy, 800 g of 50% alloy

26. **TRAVEL**  An airplane flies 1000 miles due east in 2 hours and 1000 miles due south in 3 hours. What is the average speed of the airplane? 400 mph

www.algebra1.com/self_check_quiz
27. **SCIENCE** Hectar is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?

28. **CAR MAINTENANCE** One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

29. **GRADES** In Ms. Martinez’s science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student’s average grade? **87**

30. **RESCUE** A fishing trawler has radioed the Coast Guard for a helicopter to pick up an injured crew member. At the time of the emergency message, the trawler is 660 kilometers from the helicopter and heading toward it. The average speed of the trawler is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the trawler? **2 h**

31. **ANIMALS** A cheetah is 300 feet from its prey. It starts to sprint toward its prey at 90 feet per second. At the same time, the prey starts to sprint at 70 feet per second. When will the cheetah catch its prey? **15 s**

32. **No; the sprinter would catch his opponent in 40 s or after he has run 328 m.**

33. **TRACK AND FIELD** A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.

34. **CAR MAINTENANCE** A car radiator has a capacity of 16 quarts and is filled with a 25% antifreeze solution. How much must be drained off and replaced with pure antifreeze to obtain a 40% antifreeze solution? **3.2 qt**

35. **TRAVEL** An express train travels 80 kilometers per hour from Ironwood to Wildwood. A local train, traveling at 48 kilometers per hour, takes 2 hours longer for the same trip. How far apart are Ironton and Wildwood? **240 km**

36. **FOOTBALL** NFL quarterbacks are rated for their passing performance by a type of weighted average as described in the formula below:  
\[ R = \left[ 50 + 2000(C + A) \right] + 8000\left( \frac{T}{X} \right) - 10,000\left( \frac{I}{X} \right) + 100(Y + A) \]  
In this formula,
- \( R \) represents the rating,
- \( C \) represents number of completions,
- \( A \) represents number of passing attempts,
- \( T \) represents the number to touchdown passes,
- \( I \) represents the number of interceptions, and
- \( Y \) represents the number of yards gained by passing.

In the 2000 season, Daunte Culpepper had 297 completions, 474 passing attempts, 33 touchdown passes, 16 interceptions, and 3937 passing yards. What was his rating for that year? **about 98.0**

36. Sample answer: How many grams of salt must be added to 40 grams of a 28% salt solution to obtain a 40% salt solution?

176 Chapter 3 Solving Linear Equations

**Enrichment, p. 190**

**Diophantine Equations**

The first great algebraist, Diophantus of Alexandria (about A.D. 300), devoted much of his work to the solving of indeterminate equations. These equations have an unlimited number of solutions. An example is \( a + b = c \).

When the coefficients of a system of indeterminate equations and you are asked to find solutions that must be integers, the equation is called a Diophantine equation. Such equations can be quite difficult to solve, often involving trial and error—read more online.

Solve each Diophantine equation by finding at least one pair of positive integers that make the equation true. Some hints are given to help you.

a. \( 2a + 3b = 12 \)
   a. First solve the equation for \( a \).
   b. Why won’t \( a \) be an even number? If \( a \) is odd, then \( b \) won’t be an integer.
37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. *See margin.*

How are scores calculated in a figure skating competition?

Include the following in your answer:

- an explanation of how a weighted average can be used to find a skating score, and
- a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.

### Maintain Your Skills

**Mixed Review** Solve each equation for the variable specified. *(Lesson 3-8)*

40. \(3t - 4 = 6t - s\), for \(t\)

\(t = \frac{5 - 4}{3}\)

41. \(a + 6 = \frac{b - 1}{4}\), for \(b\)

\(b = 4a + 25\)

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent. *(Lesson 3-7)*

42. original: 25

new: 14

decrease: 44%

43. original: 35

new: 42

increase: 20%

44. original: 244

new: 300

increase: 23%

45. If the probability that an event will occur is \(\frac{2}{3}\), what are the odds that the event will occur? *(Lesson 2-6)*

2:1

**Simplify each expression. (Lesson 2-3)*

46. \((2b)(-3a)\)

\(-6ab\)

47. \(3x(-3y) + (-6x)(-2y)\)

\(3xy\)

48. \(5s(-6t) + 2s(-8t)\)

\(-46st\)

**Name the set of numbers graphed. (Lesson 2-1)*

49. \([-\ldots, -2, -1, 0, 1, 2, 3]\)

50. \([0, 2, 5, 6, 8]\)

**WebQuest Internet Project**

**Can You Fit 100 Candles on a Cake?**

It’s time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

www.algebra1.com/webquest

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**Open-Ended Assessment**

**Writing** Ask students whether any of their teachers use weighted averages to calculate student grades. For example, a teacher might count homework assignments once, tests twice, and final exams three times. Have students describe the weighted average systems used by their teachers and whether the weighted averages help or hurt their scores.

**Assessment Options**

Quiz (Lessons 3-8 and 3-9) is available on p. 206 of the Chapter 3 Resource Masters.

**Answer**

37. A weighted average is used to determine a skater’s average. Answers should include the following.

- The score of the short program is added to twice the score of the long program. The sum is divided by 3.

\[
\frac{4.9 + 2(5.2)}{1 + 2} = 5.1
\]
Spreadsheet Investigation

A Follow-Up of Lesson 3-9

Getting Started

Objective Find a weighted average using a computer spreadsheet.

Materials computer spreadsheet software

Teach

- Suggest that students work in pairs, with students taking turns reading aloud the data while the partner types in the data.
- The spreadsheet on the student page shows the formulas to calculate the income. Instead of typing the cell name, such as B4, students can click on the cell.
- In order for the spreadsheet to calculate the formulas correctly, students should type an equals sign at the beginning of any cell that contains a calculation formula.

Assess

Exercises 1–4 After students work these exercises, ask them what advantages computer spreadsheets have over pen and paper calculations.

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

Finding a Weighted Average

You can use a computer spreadsheet program to calculate weighted averages. A spreadsheet allows you to make calculations and print almost anything that can be organized in a table.

The basic unit in a spreadsheet is called a cell. A cell may contain numbers, words, or a formula. Each cell is named by the column and row that describe its location. For example, cell B4 is in column B, row 4.

Example

Greta Norris manages the Java Roaster Coffee Shop. She has entered the price per pound and the number of pounds sold in October for each type of coffee in a spreadsheet. What was the average price per pound of coffee sold?

The spreadsheet shows the formula that will calculate the weighted average. The formula multiplies the price of each product by its volume and calculates its sum for all the products. Then it divides that value by the sum of the volume for all products together. To the nearest cent, the weighted average of a pound of coffee is $11.75.

1. It increases by $1.00.
2. It increases by 10%.

Exercises

For Exercises 1–4, use the spreadsheet of coffee prices.

1. What is the average price of a pound of coffee for the November sales shown in the table at the right? $11.79
2. How does the November weighted average change if all of the coffee prices are increased by $1.00?
3. How does the November weighted average change if all of the coffee prices are increased by 10%?
4. Find the weighted average of a pound of coffee if the shop sold 50 pounds of each type of coffee. How does the weighted average compare to the average of the per-pound coffee prices? Explain. See margin.

Answer

4. The average of the prices per pound is the same as the weighted average if the same number of pounds of each type are sold. This is because each price is multiplied by the same weight, and then that weight is divided out.
Choose the correct term to complete each sentence.

1. According to the (Addition, Multiplication) Property of Equality, if \(a = b\), then \(a + c = b + c\).
2. A (means, ratio) is a comparison of two numbers by division.
3. A rate is the ratio of two measurements with (the same, different) units of measure.
4. The first step in the four-step problem-solving plan is to (explore, solve) the problem.
5. \(2x + 1 = 2x + 1\) is an example of a(n) (identity, formula).
6. An equivalent equation for \(3x + 5 = 7\) is \(3x = 2, 3x = 12\).
7. If the original amount was 80 and the new amount is 90, then the percent of (decrease, increase) is 12.5%.
8. (Defining the variable, Dimensional analysis) is the process of carrying units throughout a computation.
9. The (weighted average, rate) of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
10. An example of consecutive integers is (8 and 9, 8 and 10).

Lesson-by-Lesson Review

3-1 Writing Equations

Concept Summary

- Variables are used to represent unknowns when writing equations.
- Formulas given in sentence form can be written as algebraic equations.

Example

Translate the following sentence into an equation.

The sum of \(x\) and \(y\) equals 2 plus two times the product of \(x\) and \(y\).

\[
\begin{align*}
\text{The sum of } x \text{ and } y & = 2 + \frac{\text{two times the product of } x \text{ and } y}{2} \\
x + y & = 2 + 2xy
\end{align*}
\]

www.algebra1.com/vocabulary_review

FOFABLES™

Have students look through the chapter to make sure they have included examples in their Foldables for each lesson of the chapter.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.
Exercises  Translate each sentence into an equation.  See Example 1 on page 120.
11. Three times a number \( n \) decreased by 21 is 57.  \( 3n - 21 = 57 \)
12. Four minus three times \( z \) is equal to \( z \) decreased by 2.
\( 4 - 3z = z - 2 \)
13. The sum of the square of \( a \) and the cube of \( b \) is 16.
\( a^2 + b^3 = 16 \)
14. Translate the equation \( 16 - 9r = r \) into a verbal sentence.  See Example 4 on pages 122 and 123.  Sixteen minus the product of 9 and a number \( r \) is equal to \( r \).

### 3–2 Solving Equations by Using Addition and Subtraction

**Concept Summary**
- **Addition Property of Equality**  For any numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( a + c = b + c \).
- **Subtraction Property of Equality**  For any numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( a - c = b - c \).

**Example**
Solve \( x - 13 = 45 \). Then check your solution.

\[
\begin{align*}
\text{Original equation} \quad & x - 13 = 45 \\
\text{Add 13 to each side.} \quad & x = 58 \\
\text{Substitute 58 for } x. \quad & 45 = 45 \\
\text{Simplify.} \quad & \text{The solution is 58.}
\end{align*}
\]

**Exercises**  Solve each equation. Then check your solution.  See Examples 1–4 on pages 129 and 130.

15. \( r - 21 = -37 \quad -16 \)
16. \( 14 + c = -5 \quad -19 \)
17. \( 27 = 6 + p \quad 21 \)
18. \( b + (-14) = 6 \quad 20 \)
19. \( d - (-1.2) = -7.3 \quad -8.5 \)
20. \( r + \left( -\frac{1}{2} \right) = -\frac{3}{4} \quad \frac{1}{4} \)

### 3–3 Solving Equations by Using Multiplication and Division

**Concept Summary**
- **Multiplication Property of Equality**  For any numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( ac = bc \).
- **Division Property of Equality**  For any numbers \( a \), \( b \), and \( c \), with \( c \neq 0 \), if \( a = b \), then \( a/c = b/c \).

**Example**
Solve \( \frac{4}{9} t = -72 \).

\[
\begin{align*}
\text{Original equation} \quad & \frac{4}{9} t = -72 \\
\text{Multiply each side by } \frac{9}{4}. \quad & t = -162 \\
\text{Simplify.} \quad & \text{The solution is } -162.
\end{align*}
\]
Solving Multi-Step Equations

Example

Solve $34 = 8 - 2t$. Then check your solution.

\[
34 = 8 - 2t \quad \text{Original equation}
\]

\[
34 - 8 = 8 - 2t - 8 \quad \text{Subtract 8 from each side.}
\]

\[
26 = -2t \quad \text{Simplify.}
\]

\[
\frac{26}{-2} = \frac{-2t}{-2} \quad \text{Divide each side by } -2.
\]

\[
-13 = t \quad \text{Simplify.}
\]

CHECK

\[
34 = 8 - 2t \quad \text{Original equation}
\]

\[
34 \pm 8 - 2(-13) \quad \text{Substitute } -13 \text{ for } t.
\]

\[
34 = 34 \checkmark \quad \text{The solution is } -13.
\]

Exercises

Solve each equation. Then check your solution.

See Examples 2–4 on page 143.

21. $6x = -42$ \hspace{1em} 22. $-7w = -49$ \hspace{1em} 23. $\frac{3}{4}n = 30$

24. $\frac{3}{5}y = -50$ \hspace{1em} 25. $\frac{5}{2}w = -25$ \hspace{1em} 26. $5 = \frac{r}{2}$

Solving Equations with the Variable on Each Side

Concept Summary

Steps for Solving Equations

Step 1 Use the Distributive Property to remove the grouping symbols.

Step 2 Simplify the expressions on each side of the equals sign.

Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.

Step 4 Simplify the expressions on each side of the equals sign.

Step 5 Use the Multiplication and/or Division Properties of Equalities to solve.
Example
Solve \( \frac{3}{4}q - 8 = \frac{1}{4}q + 9 \).

\[
\begin{align*}
\frac{3}{4}q - 8 &= \frac{1}{4}q + 9 & \text{Original equation} \\
\frac{3}{4}q - 8 - \frac{1}{4}q &= \frac{1}{4}q + 9 - \frac{1}{4}q & \text{Subtract } \frac{1}{4}q \text{ from each side.} \\
\frac{1}{2}q - 8 &= 9 & \text{Simplify.} \\
\frac{1}{2}q + 8 &= 9 + 8 & \text{Add 8 to each side.} \\
\frac{1}{2}q &= 17 & \text{Simplify.} \\
q &= 34 & \text{Multiply each side by 2.}
\end{align*}
\]

The solution is 34.

Exercises
Solve each equation. Then check your solution.
See Examples 1–4 on pages 149 and 150.

33. \( n - 2 = 4 - 2n \) \hspace{1cm} 34. \( 3t - 2(t + 3) = t \) \hspace{1cm} 35. \( \frac{3}{6}y = 2 + \frac{1}{6}y \)

36. \( \frac{x - 2}{6} = \frac{x}{2} - 1 \) \hspace{1cm} 37. \( 2(b - 3) = 3(b - 1) \) \hspace{1cm} 38. \( 8.3h - 2.2 = 6.1h - 8.8 \)

3-6
Ratios and Proportions

Concept Summary
- A ratio is a comparison of two numbers by division.
- A proportion is an equation stating that two ratios are equal.
- A proportion can be solved by finding the cross products.

If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).

Example
Solve the proportion \( \frac{8}{7} = \frac{a}{1.75} \).

\[
\begin{align*}
\frac{8}{7} &= \frac{a}{1.75} & \text{Original equation} \\
8(1.75) &= 7(a) & \text{Find the cross products.} \\
14 &= 7a & \text{Simplify.} \\
\frac{14}{7} &= \frac{7a}{7} & \text{Divide each side by 7.} \\
2 &= a & \text{Simplify.}
\end{align*}
\]

Exercises
Solve each proportion. See Example 3 on page 156.

39. \( \frac{6}{15} = \frac{n}{45} \) \hspace{1cm} 40. \( \frac{x}{11} = \frac{35}{55} \) \hspace{1cm} 41. \( \frac{12}{d} = \frac{20}{15} \)

42. \( \frac{14}{20} = \frac{21}{m} \) \hspace{1cm} 43. \( \frac{2}{3} = \frac{h + 5}{9} \) \hspace{1cm} 44. \( \frac{6}{8} = \frac{9}{s - 4} \)
### Percent of Change

#### Concept Summary
- The proportion \( \frac{\text{amount of change}}{\text{original amount}} = \frac{r}{100} \) is used to find percents of change.

#### Example

Find the percent of change.  
original: $120  
new: $114

First, subtract to find the amount of change.  
$120 - $114 = $6  
Note that since the new amount is less than the original, the percent of change will be a percent of decrease.

Then find the percent using the original number, 120, as the base.

\[
\frac{\text{change}}{\text{original amount}} \rightarrow  \frac{6}{120} = \frac{r}{100} \quad \text{Percent proportion}
\]

\[
6(100) = 120(r) \quad \text{Find the cross products.}
\]

\[
600 = 120r \quad \text{Simplify.}
\]

\[
\frac{600}{120} = 120r \quad \text{Divide each side by 120.}
\]

\[
5 = r \quad \text{Simplify.}
\]

The percent of decrease is 5%.

#### Exercises
State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.  
See Example 1 on page 160.

45. original: 40  new: 32  dec.; 20%
46. original: 50  new: 88  inc.; 76%
47. original: 35  new: 37.1  inc.; 6%
48. Find the total price of a book that costs $14.95 plus 6.25% sales tax.  
See Example 3 on page 161.  \$15.88
49. A T-shirt priced at $12.99 is on sale for 20% off. What is the discounted price?  
See Example 4 on page 161.  \$10.39

### Solving Equations and Formulas

#### Concept Summary
- For equations with more than one variable, you can solve for one of the variables by using the same steps as solving equations with one variable.

#### Example

Solve \( \frac{x + y}{b} = c \) for \( x \).

\[
\frac{x + y}{b} = c \quad \text{Original equation}
\]

\[
\frac{x + y}{b} = c \quad \text{Multiply each side by } b.
\]

\[
x + y = bc \quad \text{Simplify.}
\]

\[
x + y - y = bc - y \quad \text{Subtract } y \text{ from each side.}
\]

\[
x = bc - y \quad \text{Simplify.}
\]
Study Guide and Review

Exercises  Solve each equation or formula for the variable specified.
See Examples 1 and 2 on pages 166 and 167.

50. $5x = y$, for $x$  \[ x = \frac{y}{5} \]

51. $ay - b = c$, for $y$  \[ y = \frac{b + c}{a} \]

52. $yx - a = cx$, for $x$  \[ x = \frac{a}{y - c} \]

53. \[ \frac{2y - a}{3} = \frac{a + 3b}{4} \], for $y$  \[ y = \frac{7a + 9b}{8} \]

3-9 Weighted Averages
Concept Summary

- The weighted average of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.
- The formula $d = rt$ is used to solve uniform motion problems.

Example

SCIENCE  Mai Lin has a 35 milliliters of 30% solution of copper sulfate. How much of a 20% solution of copper sulfate should she add to obtain a 22% solution?

Let $x$ = amount of 20% solution to be added. Make a table.

<table>
<thead>
<tr>
<th>Amount of Solution (mL)</th>
<th>Amount of Copper Sulfate</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% Solution</td>
<td>35</td>
</tr>
<tr>
<td>20% Solution</td>
<td>$x$</td>
</tr>
<tr>
<td>22% Solution</td>
<td>35 + $x$</td>
</tr>
</tbody>
</table>

\[
0.30(35) + 0.20x = 0.22(35 + x)
\]

Write and solve an equation.

10.5 + 0.20$x$ = 7.7 + 0.22$x$

Distributive Property

10.5 + 0.20$x$ - 0.20$x$ = 7.7 + 0.22$x$ - 0.20$x$

Subtract 0.20$x$ from each side.

10.5 = 7.7 + 0.02$x$

Simplify.

10.5 - 7.7 = 7.7 + 0.02$x$ - 7.7

Subtract 7.7 from each side.

2.8 = 0.02$x$

Simplify.

\[
\frac{2.8}{0.02} = \frac{0.02}{0.02}
\]

Divide each side by 0.02.

140 = $x$

Simplify.

Mai Lin should add 140 milliliters of the 20% solution.

Exercises

54. COFFEE  Ms. Anthony wants to create a special blend using two coffees, one priced at $8.40 per pound and the other at $7.28 per pound. How many pounds of the $7.28 coffee should she mix with 9 pounds of the $8.40 coffee to sell the mixture for $7.95 per pound?  \[ \text{See Example 1 on page 171.} \]

55. TRAVEL  Two airplanes leave Dallas at the same time and fly in opposite directions. One airplane travels 80 miles per hour faster than the other. After three hours, they are 2940 miles apart. What is the speed of each airplane?  \[ \text{See Example 3 on pages 172 and 173.} \]

184 Chapter 3  Solving Linear Equations
Choose the correct term to complete each sentence.

1. The study of numbers and the relationships between them is called (consecutive, number) theory.
2. An equation that is true for (every, only one) value of the variable is called an identity.
3. When a new number is (greater than, less than) the original number, the percent of change is called a percent of increase.

Translate each sentence into an equation.

4. The sum of twice x and three times y is equal to thirteen. \(2x + 3y = 13\)
5. Two thirds of a number is negative eight fifths. \(\frac{2}{3}n = -\frac{8}{5}\)

Solve each equation. Then check your solution.

6. \(-15 + k = 8\) 23
7. \(-1.2x = 7.2\) -6
8. \(k - 16 = -21\) -5

9. \(\frac{t - 7}{4} = 11\) 51
10. \(\frac{3}{4}y = -27\) -36
11. \(-12 = 7 - \frac{y}{3}\) 57

12. \(t - (-3.4) = -5.3\) -8.7
13. \(-3(x + 5) = 8x + 18\) -3
14. \(5w = 125\) 25

15. \(\frac{r}{5} - 3 = \frac{2r}{5} + 16\) -95
16. \(0.1r = 19\) 190
17. \(-\frac{2}{3}x = -\frac{4}{9}\) 2

18. \(-w + 11 = 4.6\) -6.4
19. \(2p + 1 = 5p - 11\) 4
20. \(25 - 7w = 46\) -3

Solve each proportion.

21. \(\frac{36}{t} = \frac{9}{11}\) 44
22. \(\frac{n}{4} = \frac{3.25}{52}\) 0.25
23. \(\frac{5}{12} = \frac{10}{x - 1}\) 25

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

24. original: 45
   new: 9 decrease; 80%
   25. original: 12
       new: 20 increase; 67%

Solve each equation or formula for the variable specified.

26. \(h = at - 0.25vt^2\), for \(a\) \(a = \frac{h + 0.25vt^2}{t}\)
27. \(a(y + 1) = b\), for \(y\) \(y = \frac{b - a}{a}\)

28. SALES Suppose the Central Perk coffee shop sells a cup of espresso for $2.00 and a cup of cappuccino for $2.50. On Friday, Destiny sold 30 more cups of cappuccino than espresso for a total of $178.50 worth of espresso and cappuccino. How many cups of each were sold?

29. BOATING The Yankee Clipper leaves the pier at 9:00 A.M. at 8 knots (nautical miles per hour). A half hour later, The River Rover leaves the same pier in the same direction traveling at 10 knots. At what time will The River Rover overtake The Yankee Clipper? 11:30 A.M.

30. STANDARDIZED TEST PRACTICE If \(\frac{3}{4}\) of \(\frac{3}{5}\) of \(\frac{x}{4}\) find the value of \(x\). B

   A) 12
   B) 6
   C) 3
   D) \(\frac{3}{2}\)

www.algebra1.com/chapter_test
Chapter 3  Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 3 Resource Masters.

**Part 1 Multiple Choice**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Bailey planted a rectangular garden that is 6 feet wide by 15 feet long. What is the perimeter of the garden? (Prerequisite Skill) C
   - A 21 ft
   - B 27 ft
   - C 42 ft
   - D 90 ft

2. Which of the following is true about 65 percent of 20? (Prerequisite Skill) C
   - A It is greater than 20.
   - B It is less than 10.
   - C It is less than 20.
   - D Can't tell from the information given

3. For a science project, Kelsey measured the height of a plant grown from seed. She made the bar graph below to show the height of the plant at the end of each week. Which is the most reasonable estimate of the plant's height at the end of the sixth week? (Lesson 1-8) B

4. WEAT predicted a 25% chance of snow. WFOR said the chance was 1 in 4. Myweather.com showed the chance of snow as \( \frac{1}{5} \), and Allweather.com listed the chance as 0.3. Which forecast predicted the greatest chance of snow? (Lesson 2-7) D
   - A WEAT
   - B WFOR
   - C Myweather.com
   - D Allweather.com

5. Amber owns a business that transfers photos to CD-ROMs. She charges her customers $24.95 for each CD-ROM. Her expenses include $575 for equipment and $0.80 for each blank CD-ROM. Which of these equations could be used to calculate her profit \( p \) for creating \( n \) CD-ROMs? (Lesson 3-1) A
   - A \( p = (24.95 - 0.8)n - 575 \)
   - B \( p = (24.95 + 0.8)n + 575 \)
   - C \( p = 24.95n - 574.2 \)
   - D \( p = 24.95n + 575 \)

6. Which of the following equations has the same solution as \( 8(x + 2) = 12 \)? (Lesson 3-4) D
   - A \( 8x + 2 = 12 \)
   - B \( x + 2 = 4 \)
   - C \( 8x = 10 \)
   - D \( 2x + 4 = 3 \)

7. Eduardo is buying pizza toppings for a birthday party. His recipe uses 8 ounces of shredded cheese for 6 servings. How many ounces of cheese are needed for 27 servings? (Lesson 3-6) C
   - A 27
   - B 32
   - C 36
   - D 162

8. The sum of \( x \) and \( \frac{1}{y} \) is 0, and \( y \) does not equal 0. Which of the following is true? (Lesson 3-8) D
   - A \( x = -y \)
   - B \( x = 0 \)
   - C \( x = 1 - y \)
   - D \( x = -\frac{1}{y} \)

**Additional Practice**

See pp. 209–210 in the Chapter 3 Resource Masters for additional standardized test practice.

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Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
Part 2 | Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Let \( x = 2 \) and \( y = -3 \). Find the value of \( \frac{x(y + 5)}{4} \). (Lesson 1-2) \(-0.5\)

10. Use the formula \( F = \frac{9}{5}C + 32 \) to convert temperatures from Celsius (C) to Fahrenheit (F). If it is \(-5^\circ\) Celsius, what is the temperature in degrees Fahrenheit? (Lesson 2-3) \(23\)

11. Darnell keeps his cotton socks folded in pairs in his drawer. Five pairs are black, 2 pairs are navy, and 1 pair is brown. In the dark, he pulls out one pair at random. What are the odds that it is black? (Lesson 2-6) \(5:3\)

12. The sum of the ages of the Kruger sisters is 39. Their ages can be represented as three consecutive integers. What is the age of the middle sister? (Lesson 3-4) \(13\)

13. On a car trip, Tyson drove 65 miles more than half the number of miles Pete drove. Together they drove 500 miles. How many miles did Tyson drive? (Lesson 3-5) \(210\) mi

14. Solve \( 7(x + 2) + 4(2x - 3) = 47 \) for \( x \). (Lesson 3-5) \(3\)

15. A bookshop sells used hardcover books with a 45% discount. The price of a book was \$22.95 when it was new. What is the discounted price for that book? (Lesson 3-7) \$12.62

Part 3 | Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- \( A \) the quantity in Column A is greater,
- \( B \) the quantity in Column B is greater,
- \( C \) the two quantities are equal, or
- \( D \) the relationship cannot be determined from the information given.

www.algebra1.com/standardized_test

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( -a )</td>
</tr>
<tr>
<td>( C )</td>
<td>(Lesson 2-1)</td>
</tr>
</tbody>
</table>

16.

<table>
<thead>
<tr>
<th>solution of ( 3x + 7 = 10 )</th>
<th>solution of ( 4y - 2 = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>(Lesson 3-4)</td>
</tr>
</tbody>
</table>

17.

<table>
<thead>
<tr>
<th>the percent of increase from $75 to $100</th>
<th>the percent of increase from $150 to $200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>(Lesson 3-7)</td>
</tr>
</tbody>
</table>

18.

19. See margin for explanation.

Part 4 | Open Ended

Record your answers on a sheet of paper.

19. Kirby’s pickup truck travels at a rate of 6 miles every 10 minutes. Nola’s SUV travels at a rate of 15 miles every 25 minutes. The speed limit on this street is 40 mph. Is either vehicle or are both vehicles exceeding the speed limit? Explain. (Lesson 3-6) *neither*

20. A chemist has one solution of citric acid that is 20% acid and another solution of citric acid that is 80% acid. She plans to mix these solutions together to make 200 liters of a solution that is 50% acid. (Lesson 3-9)

a. Complete the table to show the liters of 20% and 80% solutions that will be used to make the 50% solution. Use \( x \) to represent the number of liters of the 80% solution that will be used to make the 50% solution.

<table>
<thead>
<tr>
<th>Liters of Solution</th>
<th>Liters of Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Solution</td>
<td>( 200 - x )</td>
</tr>
<tr>
<td>80% Solution</td>
<td>( 0.20(200 - x) )</td>
</tr>
<tr>
<td>50% Solution</td>
<td>( x )</td>
</tr>
<tr>
<td></td>
<td>( 0.80x )</td>
</tr>
<tr>
<td></td>
<td>( 0.50(200) )</td>
</tr>
</tbody>
</table>

b. Write an equation that represents the number of liters of acid in the solution. \( 0.20(200 - x) + 0.80x = 0.50(200) \)

c. How many liters of the 20% solution and how many of the 80% solution will the chemist need to mix together to make 200 liters of a 50% solution?

100 L of 20%, 100 L of 80%

Evaluating Open-Ended Assessment Questions

Open-Ended Assessment questions are graded by using a multilevel rubric that guides you in assessing a student’s knowledge of a particular concept.

**Goal:** Analyze rate data to determine vehicle speed and travel time.

**Sample Scoring Rubric:** The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.

<table>
<thead>
<tr>
<th>Score</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>A correct solution that is supported by well-developed, accurate explanations</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no supporting evidence or explanation</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given</td>
</tr>
</tbody>
</table>

Answer

19. When you calculate the miles per hour rate for each vehicle, both the pickup and the SUV are traveling at 36 mph. Therefore, neither is exceeding the speed limit.